

Answer (u).

We need to compute  $x * h$ , where

$$x * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$x(t) = \begin{cases} 2-t & 1 \leq t < 2 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad h(t) = \begin{cases} -t-2 & -3 \leq t < -2 \\ 0 & \text{otherwise} \end{cases}$$

First, we plot  $x(\tau)$  and  $h(t-\tau)$  versus  $\tau$  in Figures (a) and (d), respectively.

Figure (e):  $t < -2$

$$x * h(t) = 0$$

Figure (f):  $-2 \leq t < -1$

$$x * h(t) = \int_1^{t+3} (2-\tau)(\tau-t-2) d\tau$$

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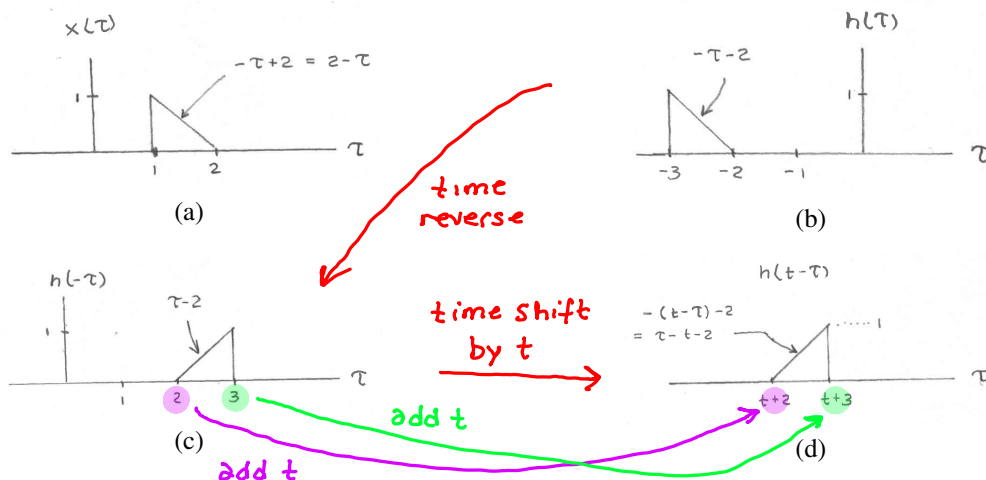
Figure (g):  $-1 \leq t < 0$

$$x * h(t) = \int_{t+2}^2 (2-\tau)(\tau-t-2) d\tau$$

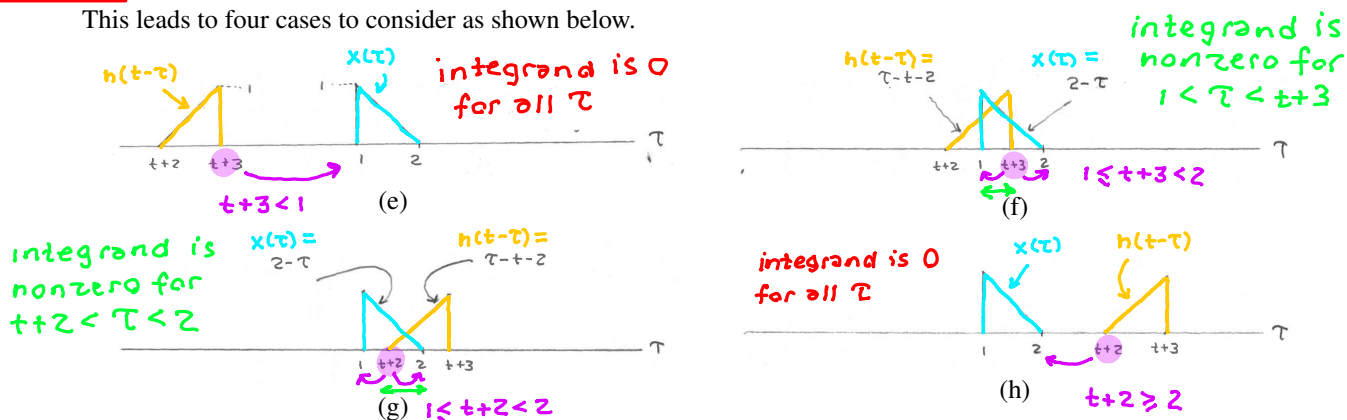
$$= \int_{t+2}^2 (2-\tau)(\tau-t-2) d\tau$$

Figure (h):  $t \geq 0$

$$x * h(t) = 0$$



This leads to four cases to consider as shown below.



From Figure (e), for  $t < -2$  (i.e.,  $t+3 < 1$ ), we have

$$x * h(t) = 0.$$

From Figure (f), for  $-2 \leq t < -1$  (i.e.,  $1 \leq t+3 < 2$ ), we have

$$x * h(t) = \int_1^{t+3} \underbrace{(2-\tau)}_{x(\tau)} \underbrace{(\tau-t-2)}_{h(t-\tau)} d\tau.$$

From Figure (g), for  $-1 \leq t < 0$  (i.e.,  $1 \leq t+2 < 2$ ), we have

$$x * h(t) = \int_{t+2}^2 \underbrace{(2-\tau)}_{x(\tau)} \underbrace{(\tau-t-2)}_{h(t-\tau)} d\tau.$$

From Figure (h), for  $t \geq 0$  (i.e.,  $t+2 \geq 2$ ), we have

$$x * h(t) = 0.$$

Simplifying, we obtain

$$x * h(t) = \begin{cases} \frac{1}{6}t^3 - t - \frac{2}{3} & -2 \leq t < -1 \\ -\frac{1}{6}t^3 & -1 \leq t < 0 \\ 0 & \text{otherwise.} \end{cases}$$