

Example 3.4. Let $x_1(t) = \cos(6\pi t)$ and $x_2(t) = \sin(30\pi t)$. Determine if the function $y = x_1 + x_2$ is periodic, and if it is, find its fundamental period.

Solution. Let T_1 and T_2 denote the fundamental periods of x_1 and x_2 , respectively. We have

$$T_1 = \frac{2\pi}{6\pi} = \frac{1}{3} \quad \text{and} \quad T_2 = \frac{2\pi}{30\pi} = \frac{1}{15}.$$

Thus, we have

$$\frac{T_1}{T_2} = \left(\frac{1}{3}\right) / \left(\frac{1}{15}\right) = \frac{15}{3} = \frac{5}{1}. \quad \rightarrow \text{5 and 1 are coprime}$$

Since $\frac{T_1}{T_2}$ is a rational number, y is periodic. Let T denote the fundamental period of y . Since 5 and 1 are coprime, we have

$$T = 1T_1 = 5T_2 = \frac{1}{3}. \quad \blacksquare$$

$$\frac{T_1}{T_2} = \frac{p}{q}$$

Cross multiplication
pattern (p, q coprime)