**Theorem 3.1** (Decomposition of function into even and odd parts)**.** *Any arbitrary function $x$ can be uniquely represented as the sum of the form*

$$x(t) = x_\mathrm{e}(t) + x_\mathrm{o}(t), \tag{3.7}$$

*where $x_\mathrm{e}$ and $x_\mathrm{o}$ are even and odd, respectively, and given by*

$$x_\mathrm{e}(t) = \tfrac{1}{2}\left[x(t) + x(-t)\right] \quad \textit{and} \tag{3.8}$$

$$x_\mathrm{o}(t) = \tfrac{1}{2}\left[x(t) - x(-t)\right]. \tag{3.9}$$

*As a matter of terminology, $x_\mathrm{e}$ is called the* ***even part*** *of $x$ and is denoted* Even$\{x\}$*, and $x_\mathrm{o}$ is called the* ***odd part*** *of $x$ and is denoted* Odd$\{x\}$.

Partial *Proof.* From (3.8) and (3.9), we can easily confirm that $x_\mathrm{e} + x_\mathrm{o} = x$ as follows:

$$\begin{aligned} x_\mathrm{e}(t) + x_\mathrm{o}(t) &= \overbrace{\tfrac{1}{2}[x(t) + x(-t)]}^{x_e} + \overbrace{\tfrac{1}{2}[x(t) - x(-t)]}^{x_o} \\ &= \tfrac{1}{2}x(t) + \tfrac{1}{2}x(-t) + \tfrac{1}{2}x(t) - \tfrac{1}{2}x(-t) \\ &= x(t). \end{aligned}$$

from the definition of $x_e$ and $x_o$

$x(-t)$ terms cancel

Furthermore, we can easily verify that $x_\mathrm{e}$ is even and $x_\mathrm{o}$ is odd. From the definition of $x_\mathrm{e}$ in (3.8), we have

$$\begin{aligned} x_\mathrm{e}(-t) &= \tfrac{1}{2}[x(-t) + x(-[-t])] \\ &= \tfrac{1}{2}[x(t) + x(-t)] \\ &= x_\mathrm{e}(t). \end{aligned}$$

substitute $-t$ for $t$ in definition of $x_e$

even

Thus, $x_\mathrm{e}$ is even. From the definition of $x_\mathrm{o}$ in (3.9), we have

$$\begin{aligned} x_\mathrm{o}(-t) &= \tfrac{1}{2}[x(-t) - x(-[-t])] \\ &= \tfrac{1}{2}[-x(t) + x(-t)] \\ &= -x_\mathrm{o}(t). \end{aligned}$$

substitute $-t$ for $t$ in definition of $x_o$

odd

Thus, $x_\mathrm{o}$ is odd.