Example 7.27. Find the inverse Laplace transform x of

$$X(s) = \frac{2}{s^2 - s - 2} \quad \text{for } -1 < \operatorname{Re}(s) < 2.$$

Solution. We begin by rewriting X in the factored form

$$X(s) = \frac{2}{(s+1)(s-2)}.$$

strictly proper with
1st order poles at −1 and 2

Then, we find a partial fraction expansion of X. We know that X has an expansion of the form

$$X(s) = \frac{A_1}{s+1} + \frac{A_2}{s-2}.$$

Calculating the coefficients of the expansion, we obtain

$$\begin{aligned}
A_1 &= (s+1)X(s)|_{s=-1} \\
&= \left.\frac{2}{s-2}\right|_{s=-1} \\
&= -\frac{2}{3} \quad \text{and} \\
A_2 &= (s-2)X(s)|_{s=2} \\
&= \left.\frac{2}{s+1}\right|_{s=2} \\
&= \frac{2}{3}.
\end{aligned}$$

So, X has the expansion

$-1 < \operatorname{Re}(s) < 2$

$$X(s) = \frac{2}{3}\left(\frac{1}{s-2}\right) - \frac{2}{3}\left(\frac{1}{s+1}\right).$$

Taking the inverse Laplace transform of both sides of this equation, we have

$$x(t) = \frac{2}{3}\underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s-2}\right\}(t)}_{①} - \frac{2}{3}\underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}(t)}_{②}. \tag{7.6}$$

LT table

Using Table 7.2 and the given ROC, we have

$-1 < \operatorname{Re}(s) < 2$

① $$-e^{2t}u(-t) \overset{\text{LT}}{\longleftrightarrow} \frac{1}{s-2} \quad \text{for } \operatorname{Re}(s) < 2 \quad \text{and}$$
② $$e^{-t}u(t) \overset{\text{LT}}{\longleftrightarrow} \frac{1}{s+1} \quad \text{for } \operatorname{Re}(s) > -1.$$

ROC must contain
$-1 < \operatorname{Re}(s) < 2$
(see Ⓐ and Ⓑ)

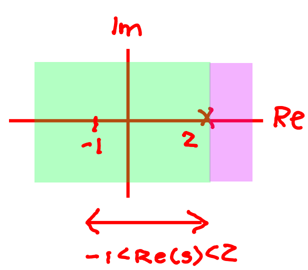
Substituting these results into (7.6), we obtain

$$\begin{aligned}
x(t) &= \tfrac{2}{3}[-e^{2t}u(-t)] - \tfrac{2}{3}[e^{-t}u(t)] \\
&= -\tfrac{2}{3}e^{2t}u(-t) - \tfrac{2}{3}e^{-t}u(t).
\end{aligned}$$

substituting the inverse LTs
from ① and ②

■

Ⓐ
possible ROCs
for $\frac{1}{s-2}$



Ⓑ
possible ROCs
for $\frac{1}{s+1}$