

Example 3.32. Determine whether the system \mathcal{H} is time invariant, where

$$\mathcal{H}x(t) = \sin[x(t)]. \quad \textcircled{1}$$

Solution. Let $x'(t) = x(t - t_0)$, where t_0 is an arbitrary real constant. From the definition of \mathcal{H} , we can easily deduce that

$$\begin{aligned} \mathcal{H}x(t - t_0) &= \sin[x(t - t_0)] && \text{by substituting } t - t_0 \text{ for } t \text{ in } \textcircled{1} \\ \mathcal{H}x'(t) &= \sin x'(t) && \text{from definition of } \mathcal{H} \text{ in } \textcircled{1} \\ &= \sin[x(t - t_0)]. && \text{from definition of } x' \text{ in } \textcircled{2} \end{aligned}$$

equal for all x and all t_0

Since $\mathcal{H}x(t - t_0) = \mathcal{H}x'(t)$ for all x and t_0 , the system is time invariant. ■

A system \mathcal{H} is said to be time invariant if, for every function x and every real constant t_0 , the following condition holds:

$$\mathcal{H}x(t - t_0) = \mathcal{H}x'(t) \text{ for all } t, \text{ where } x'(t) = x(t - t_0)$$