Example 5.6. Consider the periodic function x with period $T = 2$ as shown in Figure 5.4. Let $\hat{x}$ denote the Fourier series representation of x (i.e., $\hat{x}(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$, where $\omega_0 = \pi$). Determine the values $\hat{x}(0)$ and $\hat{x}(1)$.

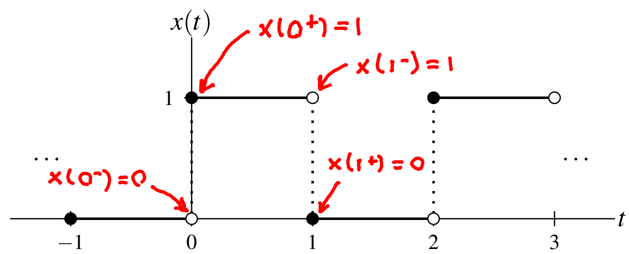

Figure 5.4: Periodic function x.

theorem for function satisfying Dirichlet conditions

Solution. We begin by observing that x satisfies the Dirichlet conditions. Consequently, Theorem 5.4 applies. Thus, we have that

$$\begin{aligned}
\hat{x}(0) &= \tfrac{1}{2}\left[x(0^-) + x(0^+)\right] \quad \leftarrow \text{average of left and right limits} \\
&= \tfrac{1}{2}(0+1) \\
&= \tfrac{1}{2} \quad \text{and} \\
\hat{x}(1) &= \tfrac{1}{2}\left[x(1^-) + x(1^+)\right] \quad \leftarrow \text{average of left and right limits} \\
&= \tfrac{1}{2}(1+0) \\
&= \tfrac{1}{2}.
\end{aligned}$$

■