Example 6.12 (Fourier transform of a real function)**.** Let X denote the Fourier transform of the function x. Show that, if x is real, then X is conjugate symmetric (i.e., $X(\omega) = X^*(-\omega)$ for all ω).

Solution. From the conjugation property of the Fourier transform, we have

$$\mathcal{F}\{x^*(t)\}(\omega) = X^*(-\omega).$$ ← from conjugation property

$x^* = x$ since x is real

Since x is real, we can replace x^* with x to yield

$$\mathcal{F}x(\omega) = X^*(-\omega),$$

$Fx = X$ (by definition)

or equivalently

$$X(\omega) = X^*(-\omega).$$ ■