Example 6.13 (Fourier transform of the sinc function)**.** Using the transform pair

$$\underbrace{\operatorname{rect} t}_{v(t)} \stackrel{\text{CTFT}}{\longleftrightarrow} \underbrace{\operatorname{sinc}\left(\frac{\omega}{2}\right)}_{V(\omega)}, \quad (1)$$

find the Fourier transform X of the function

$$x(t) = \operatorname{sinc}\left(\frac{t}{2}\right).$$

Solution. From the given Fourier transform pair, we have

$$v(t) = \operatorname{rect} t \quad \stackrel{\text{CTFT}}{\longleftrightarrow} \quad V(\omega) = \operatorname{sinc}\left(\frac{\omega}{2}\right). \quad \leftarrow \text{simply restating given FT pair (1)}$$

By duality, we have $\mathcal{F}V(\omega) = 2\pi v(-\omega)$

$$V(t) = \operatorname{sinc}\left(\frac{t}{2}\right) \quad \stackrel{\text{CTFT}}{\longleftrightarrow} \quad \mathcal{F}V(\omega) \underset{\text{duality}}{=} 2\pi v(-\omega) \underset{\text{given FT pair (1)}}{=} 2\pi \operatorname{rect}(-\omega) \underset{\text{rect is even}}{=} 2\pi \operatorname{rect} \omega.$$

Thus, we have

$$V(t) = \operatorname{sinc}\left(\frac{t}{2}\right) \quad \stackrel{\text{CTFT}}{\longleftrightarrow} \quad \mathcal{F}V(\omega) = 2\pi \operatorname{rect} \omega.$$

Observing that $V = x$ and $\mathcal{F}V = X$, we can rewrite the preceding relationship as

$$x(t) = \operatorname{sinc}\left(\frac{t}{2}\right) \quad \stackrel{\text{CTFT}}{\longleftrightarrow} \quad X(\omega) = 2\pi \operatorname{rect} \omega.$$

Thus, we have shown that

$$X(\omega) = 2\pi \operatorname{rect} \omega. \qquad \blacksquare$$