Example 7.11 (Laplace-domain shifting property). Using only the properties of the Laplace transform and the transform pair

$$e^{-|t|} \overset{\text{LT}}{\longleftrightarrow} \frac{2}{1-s^2} \quad \text{for } -1 < \operatorname{Re}(s) < 1,$$

find the Laplace transform X of

$$x(t) = e^{5t}e^{-|t|}.$$

Solution. We are given

$$e^{-|t|} \overset{\text{LT}}{\longleftrightarrow} \frac{2}{1-s^2} \quad \text{for } -1 < \operatorname{Re}(s) < 1.$$

shift ROC by 5

Using the Laplace-domain shifting property, we can deduce

shift s by 5

multiply by e^{5t}

$$x(t) = e^{5t}e^{-|t|} \overset{\text{LT}}{\longleftrightarrow} X(s) = \frac{2}{1-(s-5)^2} \quad \text{for } \underbrace{-1+5}_{4} < \operatorname{Re}(s) < \underbrace{1+5}_{6},$$

Thus, we have

$$X(s) = \frac{2}{1-(s-5)^2} \quad \text{for } 4 < \operatorname{Re}(s) < 6.$$

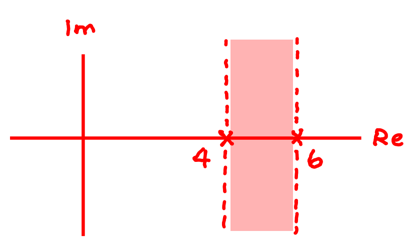
not strictly necessary except to check answer

Rewriting X in factored form, we have

$$X(s) = \frac{2}{1-(s-5)^2} = \frac{2}{1-(s^2-10s+25)} = \frac{2}{-s^2+10s-24} = \frac{-2}{s^2-10s+24} = \frac{-2}{(s-6)(s-4)}.$$

Therefore, we have

$$X(s) = \frac{-2}{(s-4)(s-6)} \quad \text{for } 4 < \operatorname{Re}(s) < 6. \qquad \blacksquare$$



Sanity check:

are stated algebraic expression and stated ROC self consistent?

yes, ROC bounded by poles