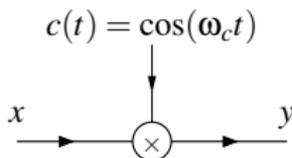


DSB-SC AM: Transmitter



$$y(t) = \cos(\omega_c t)x(t)$$

$$X = \mathcal{F}x, \quad Y = \mathcal{F}y$$

use modulation property
not multiplication property!

$$Y(\omega) = \mathcal{F}\{\cos(\omega_c t)x(t)\}(\omega)$$

$$= \mathcal{F}\left\{\frac{1}{2}(e^{j\omega_c t} + e^{-j\omega_c t})x(t)\right\}(\omega)$$

$$= \frac{1}{2}[\mathcal{F}\{e^{j\omega_c t}x(t)\}(\omega) + \mathcal{F}\{e^{-j\omega_c t}x(t)\}(\omega)]$$

$$= \frac{1}{2}[X(\omega - \omega_c) + X(\omega + \omega_c)]$$

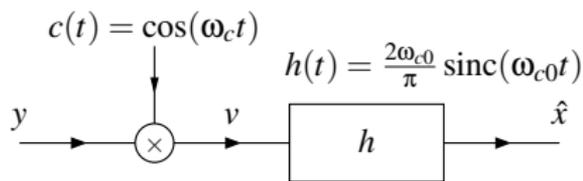
take FT

Euler

linearity
property

modulation
property

DSB-SC AM: Receiver



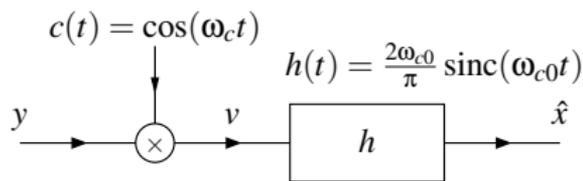
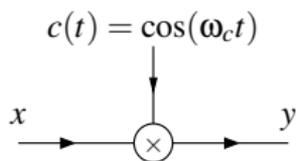
$$v(t) = \overset{\textcircled{1}}{\cos(\omega_c t)} y(t), \quad h(t) = \frac{2\omega_{c0}}{\pi} \overset{\textcircled{2}}{\text{sinc}(\omega_{c0} t)}, \quad \hat{x}(t) = v * \overset{\textcircled{3}}{h(t)}$$

use modulation property, not multiplication property!!!

$$Y = \mathcal{F}y, \quad V = \mathcal{F}v, \quad H = \mathcal{F}h, \quad \hat{X} = \mathcal{F}\hat{x}$$

$$\begin{aligned} V(\omega) &= \mathcal{F}\{\cos(\omega_c t)y(t)\}(\omega) \quad \leftarrow \text{FT of } \textcircled{1} \\ &= \mathcal{F}\left\{\frac{1}{2}(e^{j\omega_c t} + e^{-j\omega_c t})y(t)\right\}(\omega) \quad \leftarrow \text{Euler} \\ &= \frac{1}{2}[\mathcal{F}\{e^{j\omega_c t}y(t)\}(\omega) + \mathcal{F}\{e^{-j\omega_c t}y(t)\}(\omega)] \quad \leftarrow \text{linearity property} \\ &= \frac{1}{2}[Y(\omega - \omega_c) + Y(\omega + \omega_c)] \quad \leftarrow \text{modulation property} \\ H(\omega) &= \mathcal{F}\left\{\frac{2\omega_{c0}}{\pi} \text{sinc}(\omega_{c0} t)\right\}(\omega) \quad \leftarrow \text{FT of } \textcircled{2} \\ &= 2 \text{rect}\left(\frac{\omega}{2\omega_{c0}}\right) \quad \leftarrow \begin{array}{l} \text{from FT table} \\ \frac{B}{\pi} \text{sinc}(Bt) \xleftrightarrow{\text{FT}} \text{rect}\left(\frac{\omega}{2B}\right) \end{array} \\ \hat{X}(\omega) &= H(\omega)V(\omega) \quad \leftarrow \text{FT of } \textcircled{3} \text{ using convolution property} \end{aligned}$$

DSB-SC AM: Complete System

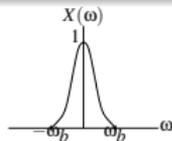


$$\textcircled{1} \quad Y(\omega) = \frac{1}{2} [X(\omega - \omega_c) + X(\omega + \omega_c)] \quad \leftarrow \text{from result for transmitter}$$

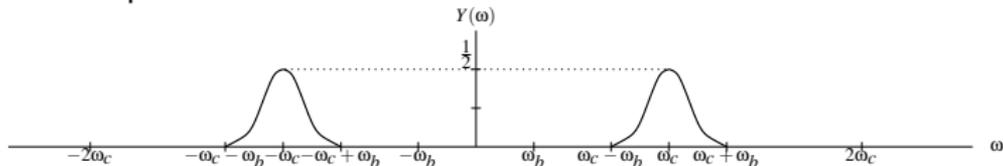
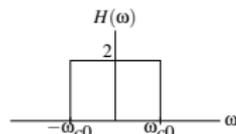
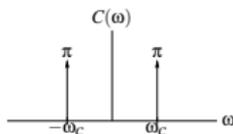
$$\begin{aligned} \textcircled{2} \quad V(\omega) &= \frac{1}{2} [Y(\omega - \omega_c) + Y(\omega + \omega_c)] \quad \leftarrow \text{from result for receiver} \\ &= \frac{1}{2} \left[\frac{1}{2} [X([\omega - \omega_c] - \omega_c) + X([\omega - \omega_c] + \omega_c)] + \right. \\ &\quad \left. \frac{1}{2} [X([\omega + \omega_c] - \omega_c) + X([\omega + \omega_c] + \omega_c)] \right] \quad \leftarrow \text{substitute } \textcircled{1} \\ &= \frac{1}{2} X(\omega) + \frac{1}{4} X(\omega - 2\omega_c) + \frac{1}{4} X(\omega + 2\omega_c) \quad \leftarrow \text{simplify} \end{aligned}$$

$$\begin{aligned} \textcircled{3} \quad \hat{X}(\omega) &= H(\omega)V(\omega) \quad \leftarrow \text{from result for receiver} \\ &= H(\omega) \left[\frac{1}{2} X(\omega) + \frac{1}{4} X(\omega - 2\omega_c) + \frac{1}{4} X(\omega + 2\omega_c) \right] \quad \leftarrow \text{substitute } \textcircled{2} \\ &= \frac{1}{2} H(\omega) X(\omega) + \frac{1}{4} H(\omega) X(\omega - 2\omega_c) + \frac{1}{4} H(\omega) X(\omega + 2\omega_c) \quad \leftarrow \text{multiply} \\ &= \frac{1}{2} [2X(\omega)] + \frac{1}{4} (0) + \frac{1}{4} (0) \quad \leftarrow \begin{array}{l} X(\omega - 2\omega_c) = 0 \text{ and } X(\omega + 2\omega_c) = 0 \text{ when } H(\omega) \neq 0 \\ \text{(since } \omega_b < \omega_c < 2\omega_c - \omega_b \text{)} \end{array} \\ &= X(\omega) \quad \leftarrow \text{simplify} \end{aligned}$$

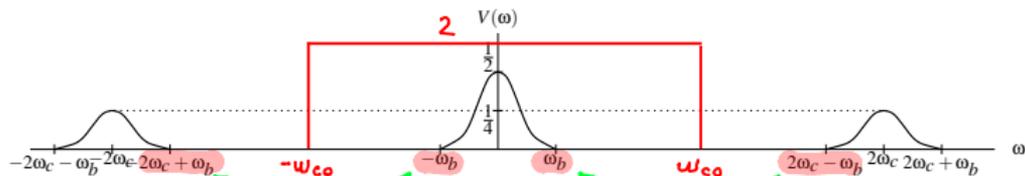
DSB-SC AM: Spectra



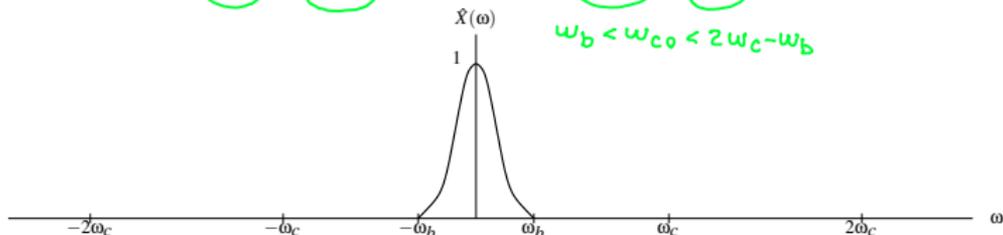
Transmitter Input



Transmitter Output



$$\omega_b < \omega_{c0} < 2\omega_c - \omega_b$$



Receiver Output