Example 6.20. Let X_1 and X_2 denote the Fourier transforms of x_1 and x_2, respectively. Suppose that X_1 and X_2 are as shown in Figures 6.6(a) and (b). Determine whether x_1 and x_2 are periodic.
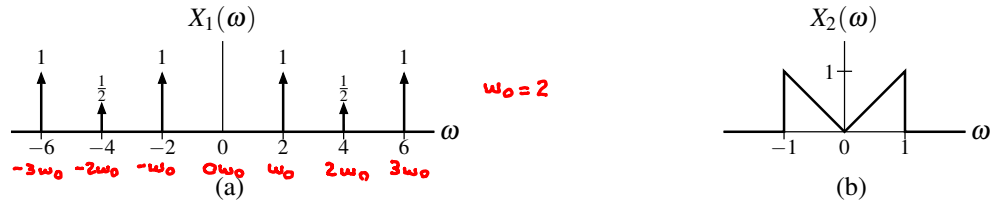

Figure 6.6: Frequency spectra. The frequency spectra (a) X_1 and (b) X_2.

Solution. We know that the Fourier transform X of a T-periodic function x must be of the form

$$X(\omega) = \sum_{k=-\infty}^{\infty} \alpha_k \delta(\omega - k\omega_0),$$

where $\omega_0 = \frac{2\pi}{T}$ and the $\{\alpha_k\}$ are complex constants. The spectrum X_1 does have this form, with $\omega_0 = 2$ and $T = \frac{2\pi}{2} = \pi$. Therefore, x_1 must be π-periodic. The spectrum X_2 does not have this form. Therefore, x_2 must not be periodic. ∎