Example 3.11 (Rectangular function)**.** Show that the rect function can be expressed in terms of u as

$$\operatorname{rect} t = u\left(t+\tfrac{1}{2}\right) - u\left(t-\tfrac{1}{2}\right).$$

Solution. Using the definition of u and time-shift transformations, we have

$$u\left(t+\tfrac{1}{2}\right) = \begin{cases} 1 & t \geq -\frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad u\left(t-\tfrac{1}{2}\right) = \begin{cases} 1 & t \geq \frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

Thus, we have

$$= \begin{cases} 0-0 & t < -\frac{1}{2} \\ 1-0 & -\frac{1}{2} \leq t < \frac{1}{2} \\ 1-1 & t \geq \frac{1}{2} \end{cases}$$

$$\begin{aligned} u\left(t+\tfrac{1}{2}\right) - u\left(t-\tfrac{1}{2}\right) &= \begin{cases} 0 & t < -\frac{1}{2} \\ 1 & -\frac{1}{2} \leq t < \frac{1}{2} \\ 0 & t \geq \frac{1}{2} \end{cases} \\ &= \begin{cases} 1 & -\frac{1}{2} \leq t < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \\ &= \operatorname{rect} t. \end{aligned}$$

Graphically, we have the scenario depicted in Figure 3.24.

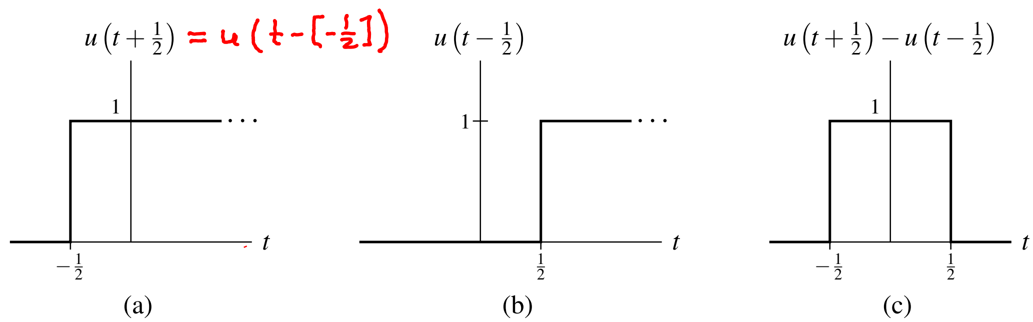

Figure 3.24: Representing the rectangular function using unit-step functions. (a) A shifted unit-step function, (b) another shifted unit-step function, and (c) their difference (which is the rectangular function).

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recall:

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$