

Example 3.32. Determine whether the system \mathcal{H} is time invariant, where

$$\mathcal{H}x(t) = \sin[x(t)]. \quad (1)$$

Solution. Let $x'(t) = x(t - t_0)$, where t_0 is an arbitrary real constant. From the definition of \mathcal{H} , we can easily deduce that

equal for all x and all t_0 \rightarrow $\mathcal{H}x(t - t_0) = \sin[x(t - t_0)]$ \leftarrow by substituting $t - t_0$ for t in (1) and $\mathcal{H}x'(t) = \sin x'(t)$ \leftarrow from definition of \mathcal{H} in (1) $= \sin[x(t - t_0)]$. \leftarrow from definition of x' in (2)

Since $\mathcal{H}x(t - t_0) = \mathcal{H}x'(t)$ for all x and t_0 , the system is time invariant. ■

A system \mathcal{H} is said to be time invariant if, for every function x and every real constant t_0 , the following condition holds:

$$\mathcal{H}x(t - t_0) = \mathcal{H}x'(t) \text{ for all } t, \text{ where } x'(t) = x(t - t_0)$$