Example 3.12 (Piecewise-linear function)**.** Consider the piecewise-linear function x given by

$$x(t) = \begin{cases} t & 0 \le t < 1 \\ 1 & 1 \le t < 2 \\ 3-t & 2 \le t < 3 \\ 0 & \text{otherwise.} \end{cases}$$

Find a single expression for $x(t)$ (involving unit-step functions) that is valid for all t.

Solution. A plot of x is shown in Figure 3.25(a). We consider each segment of the piecewise-linear function separately. The first segment (i.e., for $0 \le t < 1$) can be expressed as

$$v_1(t) = t[u(t) - u(t-1)].$$

This function is plotted in Figure 3.25(b). The second segment (i.e., for $1 \le t < 2$) can be expressed as

$$v_2(t) = [u(t-1) - u(t-2)]\ (1)$$

This function is plotted in Figure 3.25(c). The third segment (i.e., for $2 \le t < 3$) can be expressed as

$$v_3(t) = (3-t)[u(t-2) - u(t-3)].$$

This function is plotted in Figure 3.25(d). Now, we observe that $x = v_1 + v_2 + v_3$. That is, we have

$$\begin{aligned} x(t) &= v_1(t) + v_2(t) + v_3(t) \\ &= t[u(t) - u(t-1)] + [u(t-1) - u(t-2)] + (3-t)[u(t-2) - u(t-3)] \\ &= tu(t) + (1-t)u(t-1) + (3-t-1)u(t-2) + (t-3)u(t-3) \\ &= tu(t) + (1-t)u(t-1) + (2-t)u(t-2) + (t-3)u(t-3). \end{aligned}$$

Thus, we have found a single expression for $x(t)$ that is valid for all t.

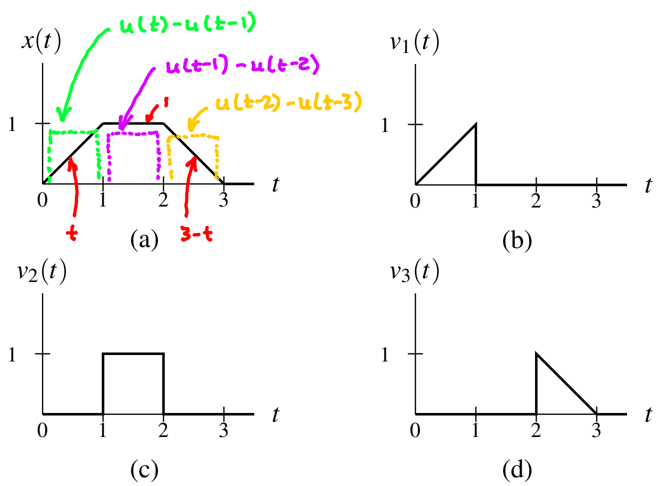

Figure 3.25: Representing a piecewise-linear function using unit-step functions. (a) The function x. (b), (c), and (d) Three functions whose sum is x.

■