Example 4.15. Consider the LTI system with input x and output y defined by

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau \quad (1)$$

(i.e., an ideal integrator). Determine whether this system is BIBO stable.

Solution. First, we find the impulse response h of the system. We have

$$\begin{aligned} h(t) &= \int_{-\infty}^{t} \delta(\tau)\,d\tau \\ &= \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases} \\ &= u(t). \end{aligned}$$

using (1) and $h = \mathcal{H}\delta$

integral is 1 if integration interval includes origin

definition of unit-step function

Using this expression for h, we now check to see if h is absolutely integrable. We have

$$\begin{aligned} \int_{-\infty}^{\infty} |h(t)|\,dt &= \int_{-\infty}^{\infty} |u(t)|\,dt \\ &= \int_{0}^{\infty} 1\,dt \\ &= \infty. \end{aligned}$$

$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$

Thus, h is not absolutely integrable. Therefore, the system is not BIBO stable. ■