Example 7.34. For each LTI system with system function H given below, determine the ROC of H that corresponds to a BIBO stable system.

(a) $H(s) = \dfrac{s(s-1)}{(s+2)(s+1+j)(s+1-j)}$;

(b) $H(s) = \dfrac{s}{(s+1)(s-1)(s-1-j)(s-1+j)}$;

(c) $H(s) = \dfrac{(s+j)(s-j)}{(s+2-j)(s+2+j)}$; and

(d) $H(s) = \dfrac{s-1}{s}$.

all rational functions

Solution. (a) The function H has poles at -2, $-1+j$, and $-1-j$. The poles are shown in Figure 7.22(a). Since H is rational, the ROC must be bounded by poles or extend to infinity. Consequently, only three distinct ROCs are possible:

i) $\operatorname{Re}(s) < -2$,
ii) $-2 < \operatorname{Re}(s) < -1$, and
iii) $\operatorname{Re}(s) > -1$.

Since we want a stable system, the ROC must include the entire imaginary axis. Therefore, the ROC must be $\operatorname{Re}(s) > -1$. This is the shaded region in the Figure 7.22(a).

(b) The function H has poles at -1, 1, $1+j$, and $1-j$. The poles are shown in Figure 7.22(b). Since H is rational, the ROC must be bounded by poles or extend to infinity. Consequently, only three distinct ROCs are possible:

i) $\operatorname{Re}(s) < -1$,
ii) $-1 < \operatorname{Re}(s) < 1$, and
iii) $\operatorname{Re}(s) > 1$.

Since we want a stable system, the ROC must include the entire imaginary axis. Therefore, the ROC must be $-1 < \operatorname{Re}(s) < 1$. This is the shaded region in Figure 7.22(b).

(c) The function H has poles at $-2+j$ and $-2-j$. The poles are shown in Figure 7.22(c). Since H is rational, the ROC must be bounded by poles or extend to infinity. Consequently, only two distinct ROCs are possible:

i) $\operatorname{Re}(s) < -2$ and
ii) $\operatorname{Re}(s) > -2$.

Since we want a stable system, the ROC must include the entire imaginary axis. Therefore, the ROC must be $\operatorname{Re}(s) > -2$. This is the shaded region in Figure 7.22(c).

(d) The function H has a pole at 0. The pole is shown in Figure 7.22(d). Since H is rational, it cannot converge at 0 (which is a pole of H). Consequently, the ROC can never include the entire imaginary axis. Therefore, the system function H can never be associated with a stable system. ■

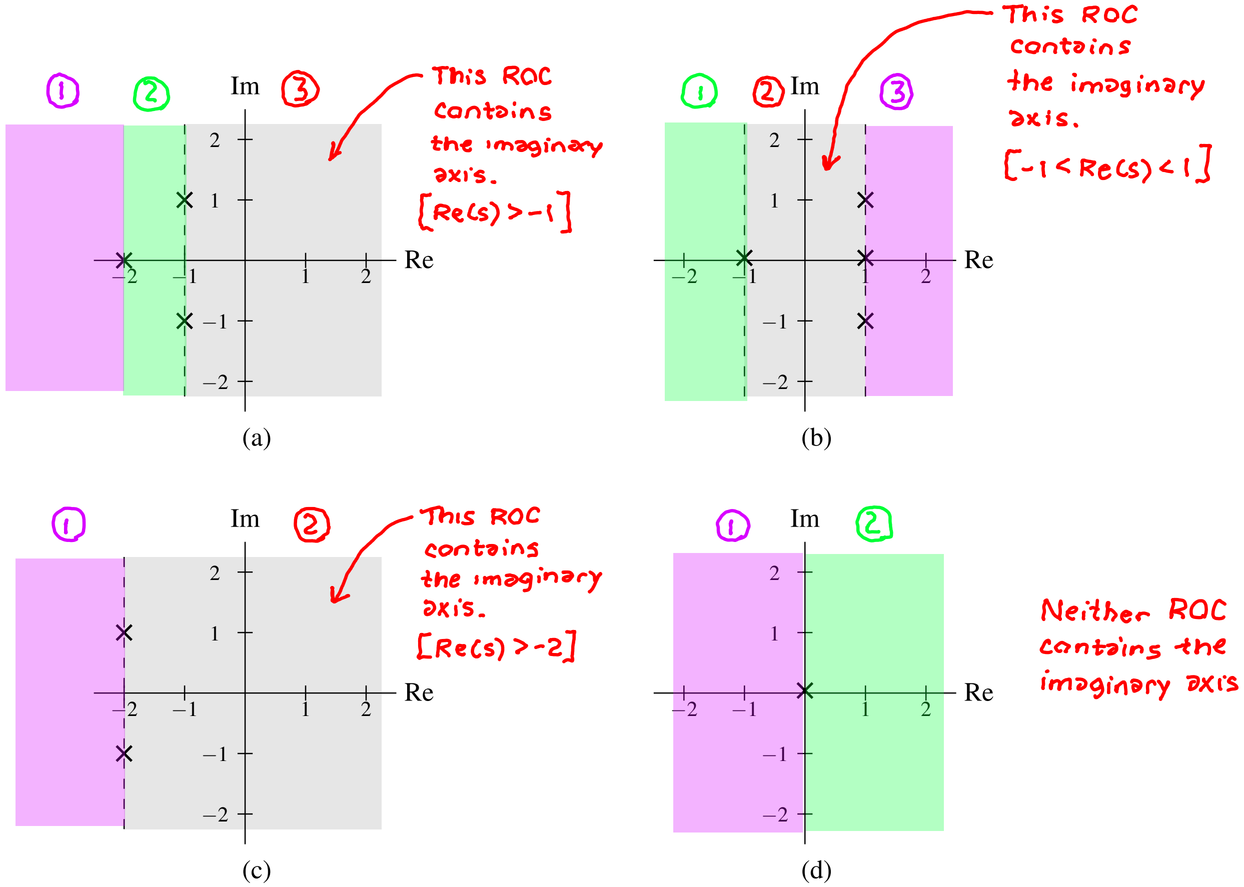

Figure 7.22: Poles and ROCs of the system function H in the (a) first, (b) second, (c) third, and (d) fourth parts of the example.