Example 3.33. Determine whether the system $\mathcal{H}$ is time invariant, where

$$\mathcal{H}x(t) = \operatorname{Odd}(x)(t) = \tfrac{1}{2}\left[x(t) - x(-t)\right]. \quad (1)$$

Solution. Let $x'(t) = x(t - t_0)$ (2), where t_0 is an arbitrary real constant. From the definition of $\mathcal{H}$, we have

$$\begin{aligned}
\mathcal{H}x(t-t_0) &= \tfrac{1}{2}[x(t-t_0) - x(-(t-t_0))] && \text{by substituting } t-t_0 \text{ for } t \text{ in (1)} \\
&= \tfrac{1}{2}[x(t-t_0) - x(-t+t_0)] \quad \text{and} \\
\mathcal{H}x'(t) &= \tfrac{1}{2}[x'(t) - x'(-t)] && \text{from definition of } \mathcal{H} \text{ in (1)} \\
&= \tfrac{1}{2}[x(t-t_0) - x(-t-t_0)]. && \text{from definition of } x' \text{ in (2)}
\end{aligned}$$

not equal if $t_0 \neq 0$

Since $\mathcal{H}x(t-t_0) = \mathcal{H}x'(t)$ does not hold for all x and t_0, the system is not time invariant. ■

only equal if $t_0 = 0$

A system $\mathcal{H}$ is said to be time invariant if, for every function x and every real constant t_0, the following condition holds:

$$\mathcal{H}x(t-t_0) = \mathcal{H}x'(t) \text{ for all } t, \text{ where } x'(t) = x(t-t_0).$$