Example 6.38 (Bandpass filtering)**.** Consider a LTI system with the impulse response

$$h(t) = \tfrac{2}{\pi}\operatorname{sinc}(t)\cos(4t).$$

from FT table:

$1 \xleftrightarrow{FT} 2\pi\delta(\omega)$

$\cos(\omega_0 t) \xleftrightarrow{FT} \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$

Using frequency-domain methods, find the response y of the system to the input

$$x(t) = -1 + 2\cos(2t) + \cos(4t) - \cos(6t).$$

Solution. Taking the Fourier transform of x, we have

taking FT

$$\begin{aligned}X(\omega) &= -2\pi\delta(\omega) + 2(\pi[\delta(\omega-2)+\delta(\omega+2)]) + \pi[\delta(\omega-4)+\delta(\omega+4)] - \pi[\delta(\omega-6)+\delta(\omega+6)] \\ &= -\pi\delta(\omega+6) + \pi\delta(\omega+4) + 2\pi\delta(\omega+2) - 2\pi\delta(\omega) + 2\pi\delta(\omega-2) + \pi\delta(\omega-4) - \pi\delta(\omega-6).\end{aligned}$$

The frequency spectrum X is shown in Figure 6.22(a). Now, we compute the frequency response H of the system. Using the results of Example 6.36, we can determine H to be

$\boxed{\text{Example 6.36 found the FT pair } \tfrac{2\omega_b}{\pi}\operatorname{sinc}(\omega_b t)\cos(\omega_a t) \xleftrightarrow{FT} \operatorname{rect}\left(\tfrac{\omega-\omega_a}{2\omega_b}\right) + \operatorname{rect}\left(\tfrac{\omega+\omega_a}{2\omega_b}\right)}$

$$\begin{aligned}H(\omega) &= \mathcal{F}\{\tfrac{2}{\pi}\operatorname{sinc}(t)\cos(4t)\}(\omega) \\ &= \operatorname{rect}\left(\tfrac{\omega-4}{2}\right) + \operatorname{rect}\left(\tfrac{\omega+4}{2}\right) \\ &= \begin{cases} 1 & 3 \le |\omega| \le 5 \\ 0 & \text{otherwise.}\end{cases}\end{aligned}$$

using result from Example 6.36 with $\omega_b = 1$, $\omega_a = 4$

definition of rect function

The frequency response H is shown in Figure 6.22(b). The frequency spectrum Y of the output is given by

$$\begin{aligned}Y(\omega) &= H(\omega)X(\omega) \\ &= \pi\delta(\omega+4) + \pi\delta(\omega-4).\end{aligned}$$

only two shifted delta functions are nonzero when $H(\omega) \neq 0$ [see Figures 6.22(a) and (b).]

Taking the inverse Fourier transform, we obtain

taking inverse FT

$$\begin{aligned}y(t) &= \mathcal{F}^{-1}\{\pi\delta(\omega+4) + \pi\delta(\omega-4)\}(t) \\ &= \mathcal{F}^{-1}\{\pi[\delta(\omega+4)+\delta(\omega-4)]\}(t) \\ &= \cos(4t).\end{aligned}$$

from table of FT pairs

$\cos(\omega_0 t) \xleftrightarrow{FT} \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$

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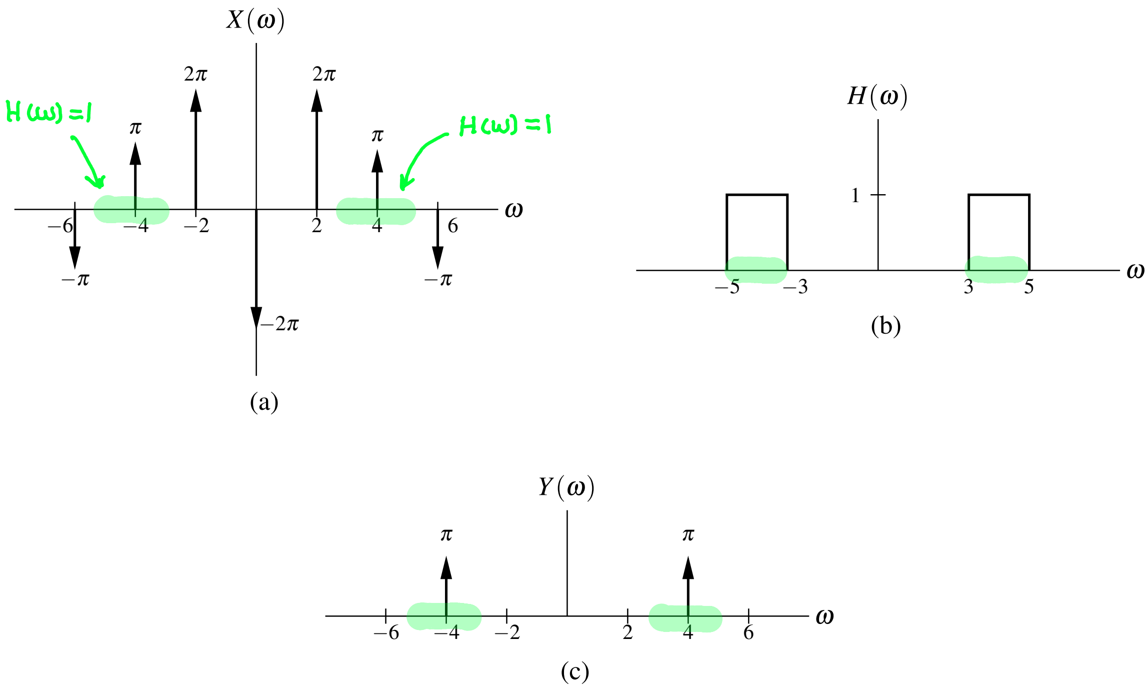

Figure 6.22: Frequency spectra for bandpass filtering example. (a) Frequency spectrum of the input x. (b) Frequency response of the system. (c) Frequency spectrum of the output y.