Example 7.8 (Linearity property of the Laplace transform)**.** Find the Laplace transform of the function

$$x = x_1 + x_2,$$

where

$$x_1(t) = e^{-t}u(t) \quad \text{and} \quad x_2(t) = e^{-t}u(t) - e^{-2t}u(t).$$

Solution. Using Laplace transform pairs from Table 7.2, we have

$$\begin{aligned}
(1)\quad X_1(s) &= \mathcal{L}\{e^{-t}u(t)\}(s) && \text{from LT table} \\
&= \frac{1}{s+1} \quad \text{for } \operatorname{Re}(s) > -1 \quad \text{and} \\
(2)\quad X_2(s) &= \mathcal{L}\{e^{-t}u(t) - e^{-2t}u(t)\}(s) && \text{linearity} \\
&= \mathcal{L}\{e^{-t}u(t)\}(s) - \mathcal{L}\{e^{-2t}u(t)\}(s) && \text{from LT table and } (*) \\
&= \frac{1}{s+1} - \frac{1}{s+2} \quad \text{for } \operatorname{Re}(s) > -1 && \text{common denominator} \\
&= \frac{1}{(s+1)(s+2)} \quad \text{for } \operatorname{Re}(s) > -1.
\end{aligned}$$

So, from the definition of X, we can write

$(*)$ $[\operatorname{Re}(s) > -2] \cap [\operatorname{Re}(s) > -1]$
$= \operatorname{Re}(s) > -1$

$$\begin{aligned}
X(s) &= \mathcal{L}\{x_1 + x_2\}(s) && \text{linearity} \\
&= X_1(s) + X_2(s) && \text{substitute expressions for } X_1 \text{ and } X_2 \text{ in (1) and (2)} \\
&= \frac{1}{s+1} + \frac{1}{(s+1)(s+2)} && \text{common denominator} \\
&= \frac{s+2+1}{(s+1)(s+2)} && \text{simplify} \\
&= \frac{s+3}{(s+1)(s+2)}.
\end{aligned}$$

but is it larger than the intersection?

Now, we must determine the ROC of X. We know that the ROC of X must contain the intersection of the ROCs of X_1 and X_2. So, the ROC must contain $\operatorname{Re}(s) > -1$. Furthermore, the ROC cannot be larger than this intersection, since X has a pole at -1. Therefore, the ROC of X is $\operatorname{Re}(s) > -1$. The various ROCs are illustrated in Figure 7.9. So, in conclusion, we have

$$X(s) = \frac{s+3}{(s+1)(s+2)} \quad \text{for } \operatorname{Re}(s) > -1. \qquad \blacksquare$$

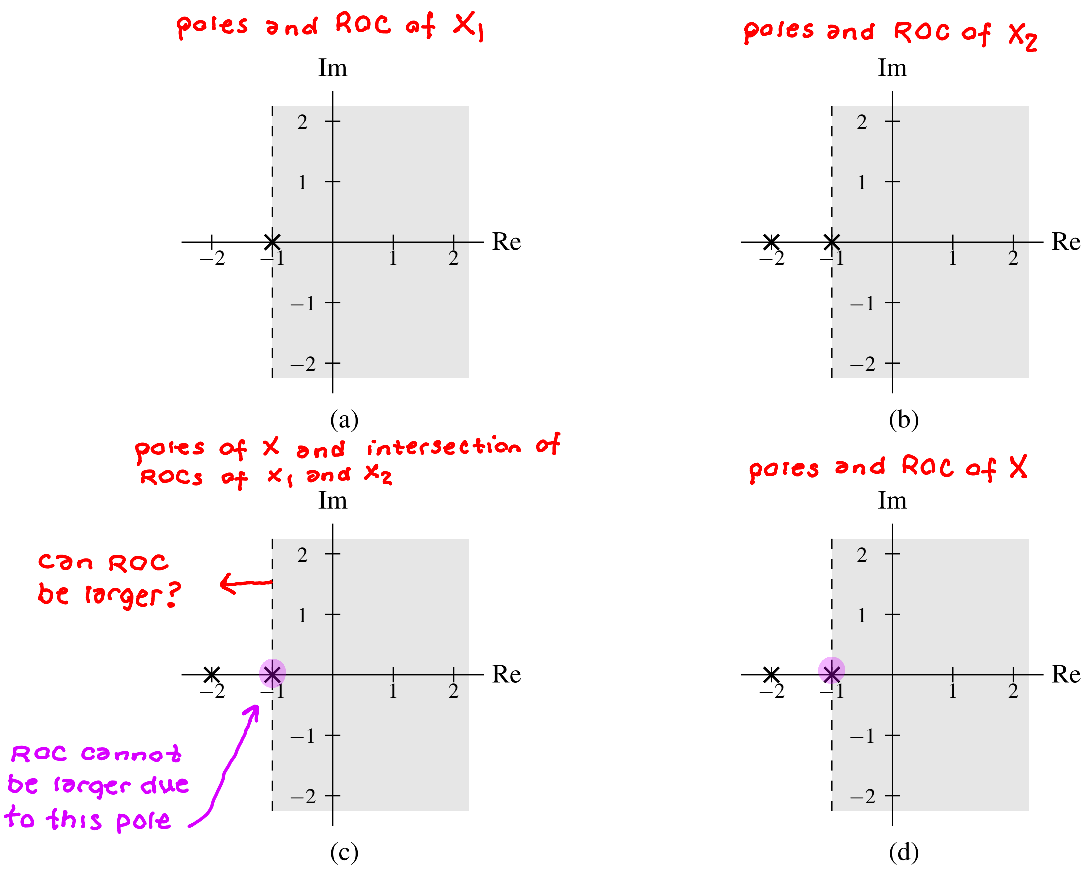

Figure 7.9: ROCs for the linearity example. The (a) ROC of X_1, (b) ROC of X_2, (c) ROC associated with the intersection of the ROCs of X_1 and X_2, and (d) ROC of X.