Example 7.31. For the LTI system with each system function H below, determine whether the system is causal.

[handwritten: rational]

(a) $H(s) = \dfrac{1}{s+1}$ for $\text{Re}(s) > -1$;

(b) $H(s) = \dfrac{1}{s^2-1}$ for $-1 < \text{Re}(s) < 1$;

[handwritten: not rational]

(c) $H(s) = \dfrac{e^s}{s+1}$ for $\text{Re}(s) < -1$; and

(d) $H(s) = \dfrac{e^s}{s+1}$ for $\text{Re}(s) > -1$.

[handwritten: causal $\Rightarrow$ ROC is RHP]

[handwritten: if rational: causal $\Leftrightarrow$ ROC is RHP]

Solution. (a) The poles of H are plotted in Figure 7.19(a) and the ROC is indicated by the shaded area. The system function H is rational and the ROC is the right-half plane to the right of the rightmost pole. Therefore, the system is causal.

(b) The poles of H are plotted in Figure 7.19(b) and the ROC is indicated by the shaded area. The system function is rational but the ROC is not a right-half plane. Therefore, the system is not causal.

(c) The system function H has a left-half plane ROC. Therefore, h is a left-sided signal. Thus, the system is not causal.

(d) The system function H has a right-half plane ROC but is not rational. Thus, we cannot make any conclusion directly from the system function. Instead, we draw our conclusion from the impulse response h. Taking the inverse Laplace transform of H, we obtain

$$h(t) = e^{-(t+1)}u(t+1).$$

[handwritten: $\leftarrow$ not causal function since $h(t) \neq 0$ for $t \in (-1,0)$]

Thus, the impulse response h is not causal. Therefore, the system is not causal.

[handwritten left: H is rational and ROC is RHP $\Rightarrow$ h is causal]

[handwritten right: H is rational and ROC is not RHP $\Rightarrow$ h is not causal]
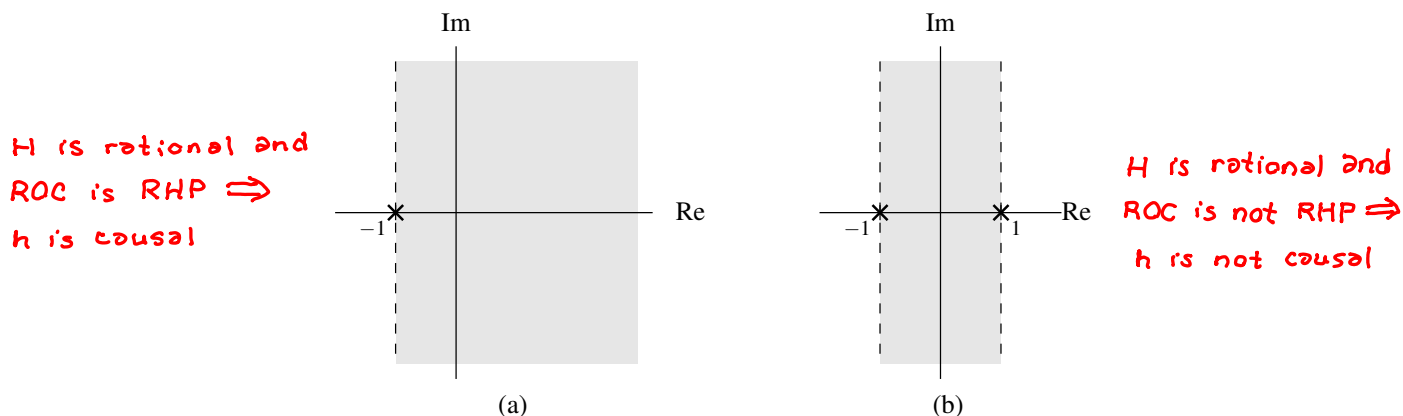

Figure 7.19: Pole and ROCs of the rational system functions in the causality example. The cases of the (a) first (b) second system functions.