Example 3.10. Evaluate the integral $\int_{-\infty}^{t}(\tau^2+1)\delta(\tau-2)d\tau$.

Solution. Using the equivalence property of the delta function given by (3.23), we can write

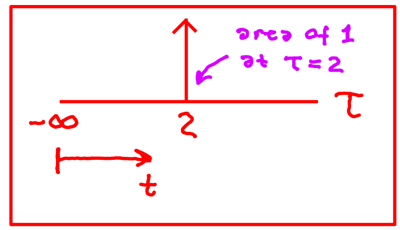
equivalence property

$$\int_{-\infty}^{t}(\tau^2+1)\delta(\tau-2)d\tau = \int_{-\infty}^{t}(2^2+1)\delta(\tau-2)d\tau$$
$$= 5\underbrace{\int_{-\infty}^{t}\delta(\tau-2)d\tau}.$$

consider simplification of the underlined integral

Using the defining properties of the delta function given by (3.22), we have that

$$\int_{-\infty}^{t}\delta(\tau-2)d\tau = \begin{cases}1 & t\geq 2\\ 0 & t<2\end{cases}$$
$$= u(t-2).$$

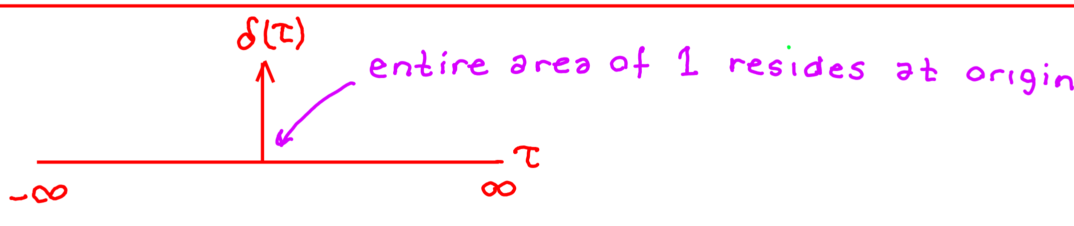


Therefore, we conclude that

$$\int_{-\infty}^{t}(\tau^2+1)\delta(\tau-2)d\tau = \begin{cases}5 & t\geq 2\\ 0 & t<2\end{cases}$$
$$= 5u(t-2).$$

$= 5\int_{-\infty}^{t}\delta(\tau-2)\,d\tau$

■



$$\int_a^b \delta(\tau)\,d\tau = \begin{cases}1 & 0\in[a,b]\\ 0 & \text{otherwise}\end{cases}$$

(where $a\leq b$)