Relationship Between the Laplace and Fourier Transforms

$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$

Recall the definition of the Laplace transform in (7.2). Consider now the special case of (7.2) where $s = j\omega$ and ω is real (i.e., $\operatorname{Re}(s) = 0$). In this case, (7.2) becomes

$$\begin{aligned} X(j\omega) &= \left[\int_{-\infty}^{\infty} x(t)e^{-st}dt\right]\bigg|_{s=j\omega} && \text{from definition of LT} \\ &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt && \text{substitute jw for s} \\ &= \mathcal{F}x(\omega). && \text{from definition of FT} \end{aligned}$$

Thus, the Fourier transform is simply the Laplace transform evaluated at $s = j\omega$, assuming that this quantity is well defined (i.e., converges). In other words,

$$X(j\omega) = \mathcal{F}x(\omega). \tag{7.4}$$

Incidentally, it is due to the preceding relationship that the Fourier transform of x is sometimes written as $X(j\omega)$. When this notation is used, the function X actually corresponds to the Laplace transform of x rather than its Fourier transform (i.e., the expression $X(j\omega)$ corresponds to the Laplace transform evaluated at points on the imaginary axis).