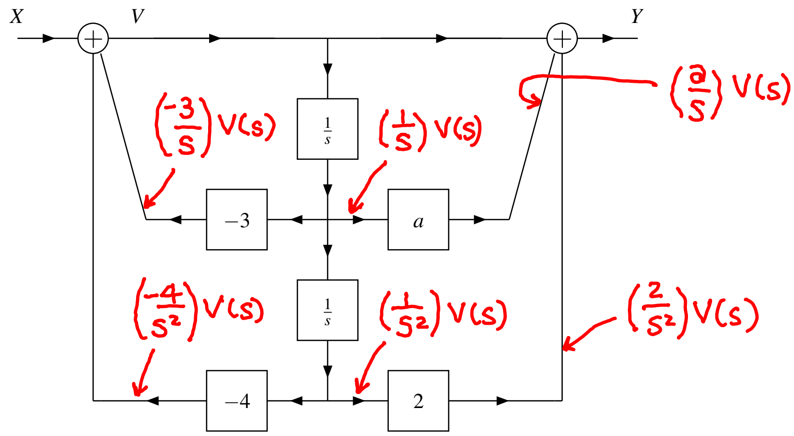
7.30 Consider the system $\mathcal{H}$ with input Laplace transform X and output Laplace transform Y as shown in the figure. In the figure, each subsystem is LTI and causal and labelled with its system function, and a is a real constant. (a) Find the system function H of the system $\mathcal{H}$. (b) Determine whether the system $\mathcal{H}$ is BIBO stable.

systematic approach to obtaining system function:
1) label system input and system output
2) label each adder output
3) write equation for each adder output and system output
4) combine equations to obtain system function



Short Answer. (a) $H(s) = \frac{s^2+as+2}{s^2+3s+4}$ for $\operatorname{Re}(s) > -\frac{3}{2}$; (b) system is BIBO stable.

Answer (a,b).
From the system block diagram, we have:

$$Y(s) = V(s) + \left(\frac{a}{s}\right)V(s) + \left(\frac{2}{s^2}\right)V(s) \quad \text{and} \quad (1)$$
$$V(s) = X(s) + \left(-\frac{3}{s}\right)V(s) + \left(-\frac{4}{s^2}\right)V(s). \quad (2)$$

The preceding two equations can be rearranged to yield

$$(3) \quad Y(s) = \left(1 + \frac{a}{s} + \frac{2}{s^2}\right)V(s) \quad \text{and} \qquad \text{rearrange (1)}$$
$$(4) \quad X(s) = \left(1 + \frac{3}{s} + \frac{4}{s^2}\right)V(s). \qquad \text{rearrange (2)}$$

divide (3) by (4)

Thus, $H(s)$ is given by

(*) $Y(s) = X(s)H(s) \Rightarrow$
$H(s) = \frac{Y(s)}{X(s)}$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1 + a/s + 2/s^2}{1 + 3/s + 4/s^2} = \frac{s^2 + as + 2}{s^2 + 3s + 4}$$

(*) rearrange

Solving for the poles of $H(s)$, we obtain

$$\frac{-3 \pm \sqrt{9 - 4(1)(4)}}{2(1)} = -\frac{3}{2} \pm \frac{j\sqrt{7}}{2}.$$

Since the poles have negative real parts, the system is BIBO stable.