Example 3.27 (Ideal integrator)**.** Determine whether the system $\mathcal{H}$ is BIBO stable, where

$$\mathcal{H}x(t) = \int_{-\infty}^{t} x(\tau)d\tau.$$

Solution. Suppose that we choose the input $x = u$ (where u denotes the unit-step function). Clearly, u is bounded (i.e., $|u(t)| \leq 1$ for all t). Calculating the response $\mathcal{H}x$ to this input, we have

$$\begin{aligned}\mathcal{H}x(t) &= \int_{-\infty}^{t} u(\tau)d\tau \\ &= \int_{0}^{t} d\tau \\ &= [\tau]|_0^t \\ &= t.\end{aligned}$$

$u(\tau) = 0$ for $\tau < 0$

From this result, however, we can see that as $t \to \infty$, $\mathcal{H}x(t) \to \infty$. Thus, the output $\mathcal{H}x$ is unbounded for the bounded input x. Therefore, the system is not BIBO stable. ■

A system $\mathcal{H}$ is said to be BIBO stable if, for every bounded function x, $\mathcal{H}x$ is bounded. That is,

$$|x(t)| \leq A < \infty \text{ for all } t \implies |\mathcal{H}x(t)| \leq B < \infty \text{ for all } t.$$

To show that a system is not BIBO stable, we simply need to find a counterexample (i.e., an example of a bounded input that yields an unbounded output).