Example 7.13 (Conjugation property)**.** Using only properties of the Laplace transform and the transform pair

$$\underbrace{e^{(-1-j)t}u(t)}_{v(t)} \overset{\text{LT}}{\longleftrightarrow} \underbrace{\frac{1}{s+1+j}}_{V(s)} \quad \text{for } \operatorname{Re}(s) > -1,$$

find the Laplace transform of

$$x(t) = e^{(-1+j)t}u(t).$$

Solution. To begin, let $v(t) = e^{(-1-j)t}u(t)$ (i.e., v is the function whose Laplace transform is given in the Laplace-transform pair above) and let V denote the Laplace transform of v. First, we determine the relationship between x and v. We have

$$\begin{aligned} x(t) &= \left(\left(e^{(-1+j)t}u(t)\right)^*\right)^* \\ &= \left(\left(e^{(-1+j)t}\right)^* u^*(t)\right)^* \\ &= \left[e^{(-1-j)t}u(t)\right]^* \\ &= v^*(t). \end{aligned}$$

$z^{**} = z$

$(z_1 z_2)^* = z_1^* z_2^*$

u is real

from definition of v

Thus, $x = v^*$. Next, we find the Laplace transform of x. We are given

$$v(t) = e^{(-1-j)t}u(t) \overset{\text{LT}}{\longleftrightarrow} V(s) = \frac{1}{s+1+j} \quad \text{for } \operatorname{Re}(s) > -1.$$

Using the conjugation property, we can deduce

conjugate

conjugate s and overall

ROC unchanged

$$x(t) = e^{(-1+j)t}u(t) \overset{\text{LT}}{\longleftrightarrow} X(s) = \left(\frac{1}{s^*+1+j}\right)^* \quad \text{for } \operatorname{Re}(s) > -1.$$

Simplifying the algebraic expression for X, we have

$$X(s) = \left(\frac{1}{s^*+1+j}\right)^* = \frac{1^*}{[s^*+1+j]^*} = \frac{1}{s+1-j}.$$

$\left(\frac{z_1}{z_2}\right)^* = \frac{z_1^*}{z_2^*}$ $\quad (z_1+z_2)^* = z_1^* + z_2^*$

Therefore, we can conclude

$$X(s) = \frac{1}{s+1-j} \quad \text{for } \operatorname{Re}(s) > -1.$$

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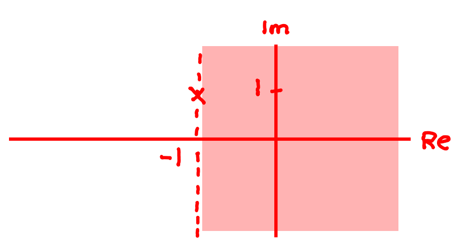

sanity check:
are the stated algebraic expression and stated ROC self consistent?
yes, the ROC is bounded by poles or extends to $\pm\infty$