Example 7.7. The Laplace transform X of the function x has the algebraic expression

$$X(s) = \frac{s+\frac{1}{2}}{(s^2+2s+2)(s^2+s-2)}.$$ ← rational function

Identify all of the possible ROCs of X. For each ROC, indicate whether the corresponding function x is left sided, right sided, two sided, or finite duration.

Solution. The possible ROCs associated with X are determined by the poles of this function. So, we must find the poles of X. Factoring the denominator of X, we obtain

$$X(s) = \frac{s+\frac{1}{2}}{(s+1-j)(s+1+j)(s+2)(s-1)}.$$

these factors obtained by using quadratic formula

Thus, X has poles at -2, $-1-j$, $-1+j$, and 1. Since these poles only have three distinct real parts (namely, -2, -1, and 1), there are four possible ROCs:

i) $\mathrm{Re}(s) < -2$,
ii) $-2 < \mathrm{Re}(s) < -1$,
iii) $-1 < \mathrm{Re}(s) < 1$, and
iv) $\mathrm{Re}(s) > 1$.

These ROCs are plotted in Figures 7.8(a), (b), (c), and (d), respectively. The first ROC is a left-half plane, so the corresponding x must be left sided. The second ROC is a vertical strip (i.e., neither a left- nor right-half plane), so the corresponding x must be two sided. The third ROC is a vertical strip (i.e., neither a left- nor right-half plane), so the corresponding x must be two sided. The fourth ROC is a right-half plane, so the corresponding x must be right sided.

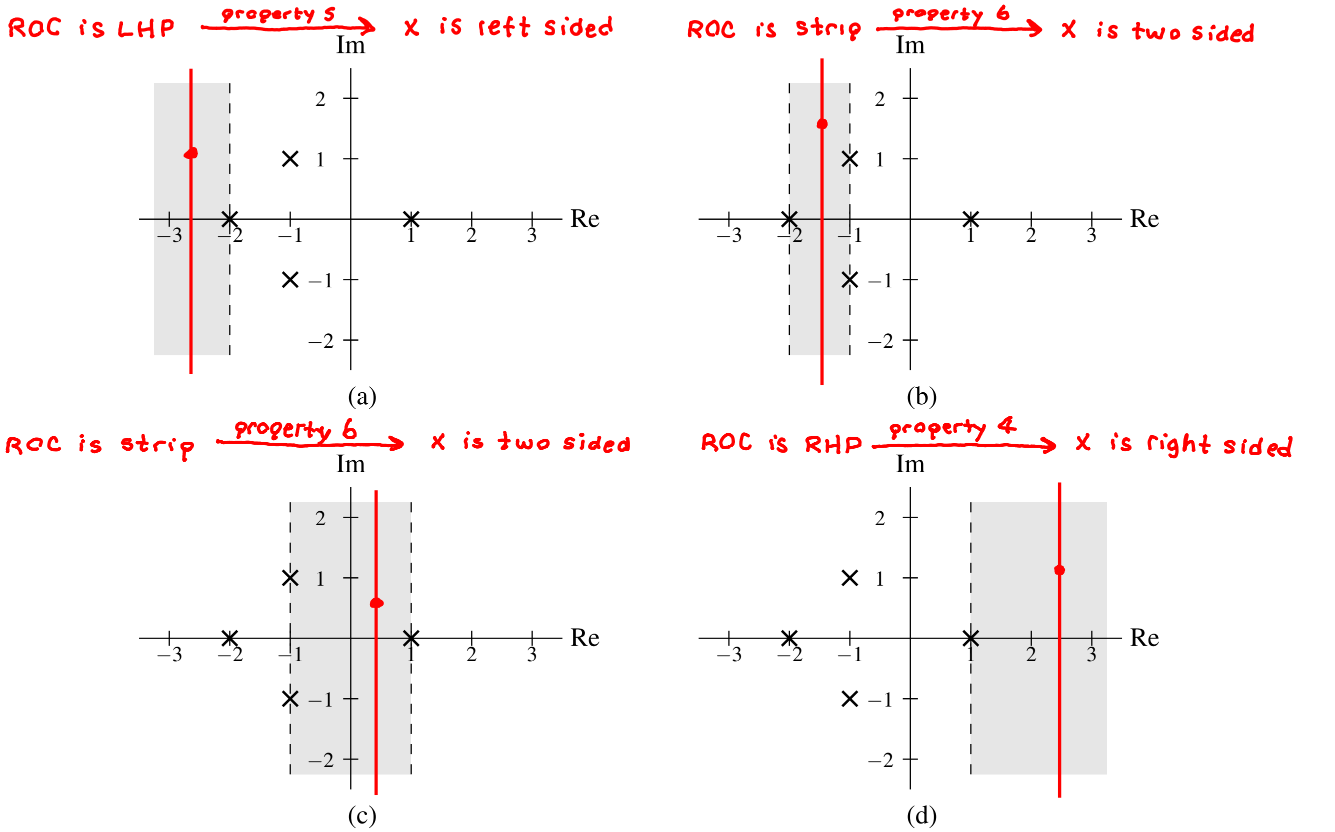

Figure 7.8: ROCs for example. The (a) first, (b) second, (c) third, and (d) fourth possible ROCs for X.

■