Example 6.21. Consider the periodic function x with fundamental period $T = 2$ as shown in Figure 6.7. Using the Fourier transform, find the Fourier series representation of x.

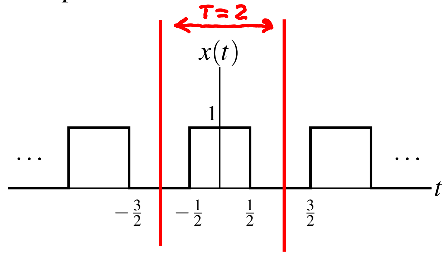

Figure 6.7: Periodic function x.

since T=2

Solution. Let ω_0 denote the fundamental frequency of x. We have that $\omega_0 = \frac{2\pi}{T} = \pi$. Let $y(t) = \operatorname{rect} t$ (i.e., y corresponds to a single period of the periodic function x). Thus, we have that

$$x(t) = \sum_{k=-\infty}^{\infty} y(t-2k).$$

Let Y denote the Fourier transform of y. Taking the Fourier transform of y, we obtain

FT table

$$Y(\omega) = (\mathcal{F}\{\operatorname{rect} t\})(\omega) = \operatorname{sinc}\left(\tfrac{1}{2}\omega\right). \qquad (1)$$

Now, we seek to find the Fourier series representation of x, which has the form

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}.$$

Using the Fourier transform, we have

$$\begin{aligned} c_k &= \tfrac{1}{T} Y(k\omega_0) && \text{sample FT of } y \text{ at } k\omega_0 \text{ for } k^{th} \text{ FS coefficient} \\ &= \tfrac{1}{2}\operatorname{sinc}\left(\tfrac{\omega_0}{2}k\right) && \text{substitute (1)} \\ &= \tfrac{1}{2}\operatorname{sinc}\left(\tfrac{\pi}{2}k\right). && \omega_0 = \pi \end{aligned}$$

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