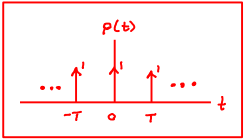
Sampling: Fourier Series for a Periodic Impulse Train



$\circledast$

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT), \quad \omega_s = \frac{2\pi}{T}$$

① $$p(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_s t}$$

p has Fourier series representation, since p is periodic

② $$\begin{aligned} c_k &= \frac{1}{T} \int_{-T/2}^{T/2} p(t) e^{-jk\omega_s t} dt \\ &= \frac{1}{T} \int_{-T/2}^{T/2} \delta(t) e^{-jk\omega_s t} dt \\ &= \frac{1}{T} \int_{-\infty}^{\infty} \delta(t) e^{-jk\omega_s t} dt \\ &= \frac{1}{T} \\ &= \frac{\omega_s}{2\pi} \end{aligned}$$

Fourier series analysis equation

see plot of p in figure $\circledast$

integrand is zero everywhere outside integration interval

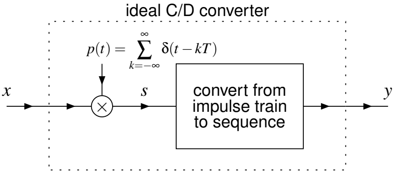
sifting property

$T = \frac{2\pi}{\omega_s}$ by definition

$$p(t) = \frac{\omega_s}{2\pi} \sum_{k=-\infty}^{\infty} e^{jk\omega_s t}$$

substitute ② into ①

Sampling: Multiplication by a Periodic Impulse Train



use modulation property, not multiplication property !!!

$$s(t) = p(t)x(t), \quad p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT), \quad \omega_s = \frac{2\pi}{T}$$

1 2

3 $$p(t) = \frac{\omega_s}{2\pi} \sum_{k=-\infty}^{\infty} e^{jk\omega_s t}$$

result of finding Fourier series representation of p in 2

$$s(t) = \frac{\omega_s}{2\pi} \sum_{k=-\infty}^{\infty} e^{jk\omega_s t} x(t)$$

substitute Fourier series representation of p in 3 into 1

$$X = \mathfrak{F}x, \quad S = \mathfrak{F}s$$

take FT using modulation property

$$S(\omega) = \frac{\omega_s}{2\pi} \sum_{k=-\infty}^{\infty} X(\omega - k\omega_s)$$