

Answer (g).

We are asked to find the Fourier transform Y of

$$y(t) = [te^{-j5t}x(t)]^*$$

In what follows, we use the **prime symbol to denote the derivative** (i.e., f' denotes the derivative of f). To begin, we have

$$\begin{aligned} y(t) &= [te^{-j5t}x(t)]^* \\ &= [e^{-j5t} \underbrace{tx(t)}^*]^* \end{aligned}$$

Letting $v_1(t) = tx(t)$, we have

$$v_1(t) = tx(t) \quad \textcircled{1}$$

$$y(t) = [e^{-j5t}v_1(t)]^*$$

Letting $v_2(t) = e^{-j5t}v_1(t)$, we have

$$v_2(t) = e^{-j5t}v_1(t) \quad \textcircled{2}$$

$$y(t) = v_2^*(t). \quad \textcircled{3}$$

Thus, we have written $y(t)$ as

$$\textcircled{3} \rightarrow y(t) = v_2^*(t)$$

where

$$\textcircled{1} \rightarrow v_1(t) = tx(t) \quad \text{and}$$

$$\textcircled{2} \rightarrow v_2(t) = e^{-j5t}v_1(t).$$

Taking the Fourier transforms of the preceding equations, we obtain

$$\begin{aligned} \textcircled{4} \quad V_1(\omega) &= jX'(\omega), && \leftarrow \text{FT of } \textcircled{1} \text{ using frequency-domain differentiation property} \\ \textcircled{5} \quad V_2(\omega) &= V_1(\omega + 5), \quad \text{and} && \leftarrow \text{FT of } \textcircled{2} \text{ using frequency-domain shifting property} \\ \textcircled{6} \quad Y(\omega) &= V_2^*(-\omega). && \leftarrow \text{FT of } \textcircled{3} \text{ using conjugation property} \end{aligned}$$

Combining the above equations, we have

$$\begin{aligned} Y(\omega) &= V_2^*(-\omega) && \leftarrow \textcircled{6} \\ &= [V_1(-\omega + 5)]^* && \leftarrow \text{substitute } \textcircled{5} \\ &= [jX'(-\omega + 5)]^* && \leftarrow \text{substitute } \textcircled{4} \\ &= -jX'^*(-\omega + 5). && \leftarrow (ab)^* = a^*b^* \end{aligned}$$