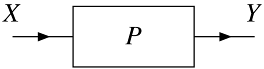
Stabilization Example: Unstable Plant

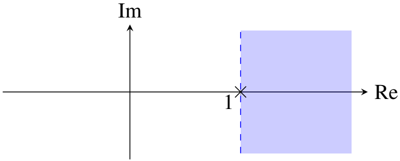
- causal LTI plant:



$$P(s) = \frac{10}{s-1}$$

has pole at 1

- ROC of P:



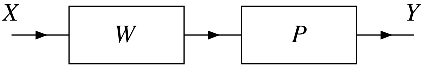
ROC is RHP since system is causal

system is not BIBO stable since ROC does not contain imaginary axis

- system is not BIBO stable

Stabilization Example: Using Pole-Zero Cancellation

- system formed by series interconnection of plant and causal LTI compensator:



$$P(s) = \frac{10}{s-1}, \quad W(s) = \frac{s-1}{10(s+1)}$$

- system function H of overall system:

pole-zero cancellation

$$H(s) = W(s)P(s) = \left(\frac{s-1}{10(s+1)}\right)\left(\frac{10}{s-1}\right) = \frac{1}{s+1}$$

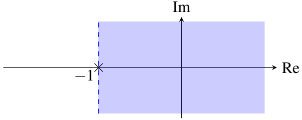
has pole at −1

connecting systems in series multiplies system functions

substitute given W and P

multiply

- ROC of H:



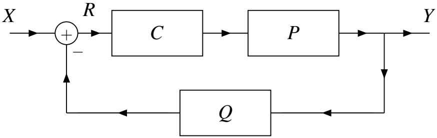
ROC is RHP since system is causal

system is BIBO stable since ROC contains imaginary axis

- overall system is BIBO stable

Stabilization Example: Using Feedback (1)

- feedback system (with causal LTI compensator and sensor):



$$P(s) = \frac{10}{s-1}, \quad C(s) = \beta, \quad Q(s) = 1$$

- system function H of feedback system:

substitute given C, P, and Q and simplify

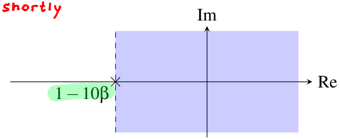
$$H(s) = \frac{C(s)P(s)}{1+C(s)P(s)Q(s)} = \frac{10\beta}{s-(1-10\beta)}$$

has pole at $1-10\beta$

we will show this shortly

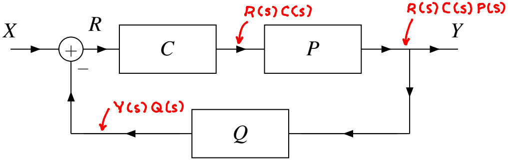
- ROC of H:

ROC contains imaginary axis if $1-10\beta<0$



ROC is RHP since system is causal

- feedback system is BIBO stable if and only if $1-10\beta < 0$ or equivalently $\beta > \frac{1}{10}$



① $$R(s) = X(s) - Q(s)Y(s)$$ equation for adder

② $$Y(s) = C(s)P(s)R(s)$$ equation for output

$$Y(s) = C(s)P(s)R(s)$$ from ②

$$= C(s)P(s)[X(s) - Q(s)Y(s)]$$ substituting formula for R from ①

$$= C(s)P(s)X(s) - C(s)P(s)Q(s)Y(s)$$ multiply

$$[1 + C(s)P(s)Q(s)]Y(s) = C(s)P(s)X(s)$$ move terms containing Y to the left-hand side and factor

$$H(s) = \frac{Y(s)}{X(s)} = \frac{C(s)P(s)}{1 + C(s)P(s)Q(s)}$$ divide both sides by X(s)[1+C(s)P(s)Q(s)]

Y(s) = X(s) H(s)

$$P(s) = \tfrac{10}{s-1}, \quad C(s) = \beta, \quad Q(s) = 1$$ given

result from previous slide

$$\begin{aligned} H(s) &= \frac{C(s)P(s)}{1+C(s)P(s)Q(s)} \\ &= \frac{\beta(\frac{10}{s-1})}{1+\beta(\frac{10}{s-1})(1)} \\ &= \frac{10\beta}{s-1+10\beta} \\ &= \frac{10\beta}{s-(1-10\beta)} \end{aligned}$$

substitute given C, P, and Q

multiply by $\frac{s-1}{s-1}$

rewrite to explicitly show pole

pole at $1-10\beta$

Remarks on Stabilization Via Pole-Zero Cancellation

- Pole-zero cancellation is not achievable in practice, and therefore it cannot be used to stabilize real-world systems.
- The theoretical models used to represent real-world systems are only approximations due to many factors, including the following:
  - Determining the system function of a system involves measurement, which always has some error.
  - A system cannot be built with such precision that it will have exactly some prescribed system function.
  - The system function of most systems will vary at least slightly with changes in the physical environment.
  - Although a LTI model is used to represent a system, the likely reality is that the system is not exactly LTI, which introduces error.
- Due to approximation error, the effective poles and zeros of the system function will only be approximately where they are expected to be.
- Since pole-zero cancellation requires that a pole and zero be placed at exactly the same location, any error will prevent this cancellation from being achieved.