table of FT pairs

Example 6.14 (Time-domain convolution property of the Fourier transform)**.** With the aid of Table 6.2, find the Fourier transform X of the function

$$x(t) = x_1 * x_2(t),$$

where

$$x_1(t) = e^{-2t}u(t) \quad \text{and} \quad x_2(t) = u(t).$$

Solution. Let X_1 and X_2 denote the Fourier transforms of x_1 and x_2, respectively. From the time-domain convolution property of the Fourier transform, we know that

$$\begin{aligned} X(\omega) &= (\mathcal{F}\{x_1 * x_2\})(\omega) \\ &= X_1(\omega)X_2(\omega). \end{aligned} \tag{6.10}$$

time-domain convolution property

table of FT pairs

From Table 6.2, we know that

① $$\begin{aligned} X_1(\omega) &= \left(\mathcal{F}\{e^{-2t}u(t)\}\right)(\omega) \\ &= \tfrac{1}{2+j\omega} \quad \text{and} \end{aligned}$$

table of FT pairs

② $$\begin{aligned} X_2(\omega) &= \mathcal{F}u(\omega) \\ &= \pi\delta(\omega) + \tfrac{1}{j\omega}. \end{aligned}$$

table of FT pairs

Substituting these expressions for $X_1(\omega)$ and $X_2(\omega)$ into (6.10), we obtain

$X(\omega) = X_1(\omega)\,X_2(\omega)$ (6.10)

substituting ① and ② into (6.10)

$$\begin{aligned} X(\omega) &= [\tfrac{1}{2+j\omega}](\pi\delta(\omega) + \tfrac{1}{j\omega}) \\ &= \tfrac{\pi}{2+j\omega}\delta(\omega) + \tfrac{1}{j\omega}(\tfrac{1}{2+j\omega}) \\ &= \tfrac{\pi}{2+j\omega}\delta(\omega) + \tfrac{1}{j2\omega-\omega^2} \\ &= \tfrac{\pi}{2}\delta(\omega) + \tfrac{1}{j2\omega-\omega^2}. \end{aligned}$$

$\left.\dfrac{\pi}{2+j\omega}\right|_{\omega=0}$

equivalence property of δ function

■