Example 3.28 (Squarer)**.** Determine whether the system $\mathcal{H}$ is BIBO stable, where

$$\mathcal{H}x(t) = x^2(t).$$

Solution. Suppose that the input x is bounded such that (for all t)

$$|x(t)| \leq A,$$

where A is a finite real constant. Squaring both sides of the inequality, we obtain

$$|x(t)|^2 \leq A^2.$$

Interchanging the order of the squaring and magnitude operations on the left-hand side of the inequality, we have

$$|x^2(t)| \leq A^2.$$

Using the fact that $\mathcal{H}x(t) = x^2(t)$, we can write

$$|\mathcal{H}x(t)| \leq A^2.$$

Since A is finite, A^2 is also finite. Thus, we have that $\mathcal{H}x$ is bounded (i.e., $|\mathcal{H}x(t)| \leq A^2 < \infty$ for all t). Therefore, the system is BIBO stable. ■

squaring a finite number always yields a finite result

A system $\mathcal{H}$ is said to be BIBO stable if, for every bounded function x, $\mathcal{H}x$ is bounded. That is,

$|x(t)| \leq A < \infty$ for all t $\Longrightarrow$ $|y(t)| \leq B < \infty$ for all t.

To show a system is BIBO stable, we must show that every bounded input produces a bounded output.