

**Example B.1** (Simple pole). Find the partial fraction expansion of the function

$$f(z) = \frac{3}{z^2 + 3z + 2}. \quad \leftarrow \text{Strictly proper}$$

*Solution.* First, we rewrite  $f$  with the denominator polynomial factored to obtain

$$f(z) = \frac{3}{(z+1)(z+2)}. \quad \leftarrow \text{Simple (i.e., 1st order) poles at } -1 \text{ and } -2$$

From this, we know that  $f$  has a partial fraction expansion of the form

$$f(z) = \frac{A_1}{z+1} + \frac{A_2}{z+2}, \quad \textcircled{1}$$

where  $A_1$  and  $A_2$  are constants to be determined. Now, we calculate  $A_1$  and  $A_2$  as follows:

$$\begin{aligned} A_1 &= (z+1)f(z)|_{z=-1} \\ &= \frac{3}{z+2} \Big|_{z=-1} \\ &= 3 \quad \text{and} \\ A_2 &= (z+2)f(z)|_{z=-2} \\ &= \frac{3}{z+1} \Big|_{z=-2} \\ &= -3. \end{aligned} \quad \left. \vphantom{\begin{aligned} A_1 \\ A_2 \end{aligned}} \right\} \textcircled{2}$$

Thus, the partial fraction expansion of  $f$  is given by

$$f(z) = \frac{3}{z+1} - \frac{3}{z+2}. \quad \leftarrow \text{from } \textcircled{1} \text{ and } \textcircled{2}$$