Example 7.14 (Time-domain convolution property)**.** Find the Laplace transform X of the function

$$x(t) = x_1 * x_2(t),$$

where

LT table

$$x_1(t) = \sin(3t)u(t) \quad \text{and} \quad x_2(t) = tu(t).$$

Solution. From Table 7.2, we have that

$$x_1(t) = \sin(3t)u(t) \overset{\text{LT}}{\longleftrightarrow} X_1(s) = \frac{3}{s^2+9} \text{ for } \operatorname{Re}(s) > 0 \quad \text{and}$$
$$x_2(t) = tu(t) \overset{\text{LT}}{\longleftrightarrow} X_2(s) = \frac{1}{s^2} \text{ for } \operatorname{Re}(s) > 0.$$

from LT table

Using the time-domain convolution property, we have

ROC equals intersection since no pole-zero cancellation

$$x_1 * x_2(t) = x(t) \overset{\text{LT}}{\longleftrightarrow} X(s) = \left(\frac{3}{s^2+9}\right)\left(\frac{1}{s^2}\right) \text{ for } \{\operatorname{Re}(s) > 0\} \cap \{\operatorname{Re}(s) > 0\}.$$

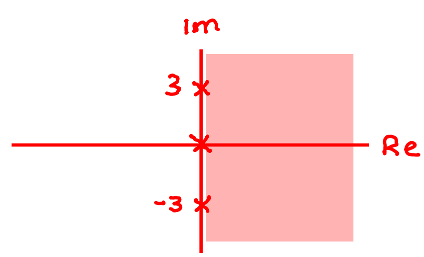
convolve

multiply

The ROC of X is $\{\operatorname{Re}(s) > 0\} \cap \{\operatorname{Re}(s) > 0\}$ (as opposed to a superset thereof), since no pole-zero cancellation occurs. Simplifying the expression for X, we conclude

$$X(s) = \frac{3}{s^2\underbrace{(s^2+9)}_{(s+3j)(s-3j)}} \text{ for } \operatorname{Re}(s) > 0. \qquad \blacksquare$$

$A \cap A = A$



sanity check:

are the stated algebraic expression and stated ROC self consistent?

yes, the ROC is bounded by poles or extends to $\pm\infty$