Example 7.42 (Unilateral Laplace transform of second-order derivative)**.** Find the unilateral Laplace transform Y of y in terms of the unilateral Laplace transform X of x, where

$$y(t) = x''(t)$$

and the prime symbol denotes derivative (e.g., x'' is the second derivative of x)

Solution. Define the function

$$v(t) = x'(t) \tag{7.17}$$

so that

$$y(t) = v'(t). \tag{7.18}$$

Let V denote the unilateral Laplace transform of v. Taking the unilateral Laplace transform of (7.17) (using the time-domain differentiation property), we have

$$\begin{aligned} V(s) &= \mathcal{L}_{\mathrm{u}}\left\{x'\right\}(s) \\ &= sX(s) - x(0^-). \end{aligned} \tag{7.19}$$

time-domain differentiation property

Taking the unilateral Laplace transform of (7.18) (using the time-domain differentiation property), we have

$$\begin{aligned} Y(s) &= \mathcal{L}_{\mathrm{u}}\left\{v'\right\}(s) \\ &= sV(s) - v(0^-). \end{aligned} \tag{7.20}$$

time-domain differentiation property

Substituting (7.19) into (7.20), we have

$$\begin{aligned} Y(s) &= s\left[sX(s) - x(0^-)\right] - v(0^-) \\ &= s^2X(s) - sx(0^-) - x'(0^-). \end{aligned}$$

substituting (7.19) into (7.20)

v = x' and multiply

Thus, we have that

$$Y(s) = s^2X(s) - sx(0^-) - x'(0^-). \qquad \blacksquare$$

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