Example 7.16 (Laplace-domain differentiation property)**.** Using only the properties of the Laplace transform and the transform pair

$$e^{-2t}u(t) \overset{\text{LT}}{\longleftrightarrow} \frac{1}{s+2} \quad \text{for } \operatorname{Re}(s) > -2,$$

find the Laplace transform of the function

$$x(t) = te^{-2t}u(t).$$

Solution. We are given

$$e^{-2t}u(t) \overset{\text{LT}}{\longleftrightarrow} \frac{1}{s+2} \quad \text{for } \operatorname{Re}(s) > -2.$$

Using the Laplace-domain differentiation and linearity properties, we can deduce

multiply by t   -d/ds   ROC unchanged

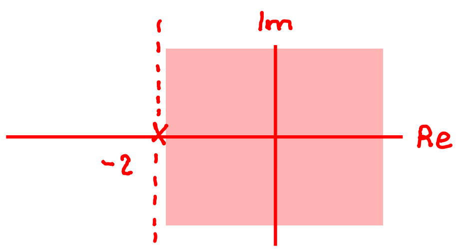
$$x(t) = te^{-2t}u(t) \quad \overset{\text{LT}}{\longleftrightarrow} \quad X(s) = -\tfrac{d}{ds}\left(\frac{1}{s+2}\right) \quad \text{for } \operatorname{Re}(s) > -2.$$

Simplifying the algebraic expression for X, we have

$$X(s) = -\tfrac{d}{ds}\left(\frac{1}{s+2}\right) = -\tfrac{d}{ds}(s+2)^{-1} = (-1)(-1)(s+2)^{-2} = \frac{1}{(s+2)^2}.$$

Therefore, we conclude

$$X(s) = \frac{1}{(s+2)^2} \quad \text{for } \operatorname{Re}(s) > -2. \qquad \blacksquare$$



Sanity check:
are the stated algebraic expression and stated ROC self consistent?

yes, the ROC is bounded by poles or extends to $\pm\infty$