Theorem 4.1 (Commutativity of convolution)**.** *Convolution is commutative. That is, for any two functions x and h,*

$$x * h = h * x. \tag{4.16}$$

In other words, the result of a convolution is not affected by the order of its operands.

Proof. We now provide a proof of the commutative property stated above. To begin, we expand the left-hand side of (4.16) as follows:

from definition of convolution

$$x * h(t) = \int_{-\infty}^{\infty} x(\tau) h(\underbrace{t-\tau}_{v}) d\tau.$$

$$\boxed{h * x(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau}$$

Next, we perform a change of variable. Let $v = t - \tau$ which implies that $\tau = t - v$ and $d\tau = -dv$. Using this change of variable, we can rewrite the previous equation as

$$\begin{aligned} x * h(t) &= \int_{t+\infty}^{t-\infty} x(t-v)h(v)(-dv) \\ &= \int_{\infty}^{-\infty} x(t-v)h(v)(-dv) \\ &= \int_{-\infty}^{\infty} x(t-v)h(v)dv \\ &= \int_{-\infty}^{\infty} h(v)x(t-v)dv \\ &= h * x(t). \end{aligned}$$

Remember that changing integration variable changes limits!

from change of variable

infinity dominates sums

$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

rearrange factors

definition of convolution

(Note that, above, we used the fact that, for any function f, $\int_a^b f(x)dx = -\int_b^a f(x)dx$.) Thus, we have proven that convolution is commutative. ■