Example 7.12 (Time-domain scaling property). Using only properties of the Laplace transform and the transform pair

$$e^{-|t|} \overset{\text{LT}}{\longleftrightarrow} \frac{2}{1-s^2} \quad \text{for } -1 < \text{Re}(s) < 1,$$

find the Laplace transform of the function

$$x(t) = e^{-|3t|}.$$

Solution. We are given

$$e^{-|t|} \overset{\text{LT}}{\longleftrightarrow} \frac{2}{1-s^2} \quad \text{for } -1 < \text{Re}(s) < 1.$$

ROC scales by 3

Using the time-domain scaling property, we can deduce

time and amplitude scale

time scale by 3

$$x(t) = e^{-|3t|} \overset{\text{LT}}{\longleftrightarrow} X(s) = \frac{1}{|3|} \frac{2}{1-(\frac{s}{3})^2} \quad \text{for } \underbrace{3(-1)}_{-3} < \text{Re}(s) < \underbrace{3(1)}_{3}.$$

Thus, we have

$$X(s) = \frac{2}{3\left[1-(\frac{s}{3})^2\right]} \quad \text{for } -3 < \text{Re}(s) < 3.$$

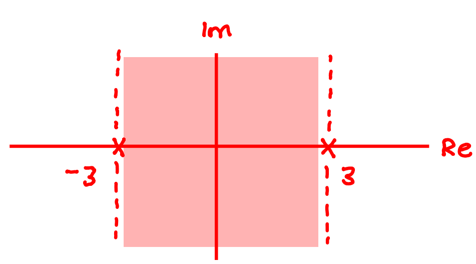
not strictly necessary except as sanity check

Simplifying, we have

$$X(s) = \frac{2}{3(1-\frac{s^2}{9})} = \frac{2}{3(\frac{9-s^2}{9})} = \frac{2(9)}{3(9-s^2)} = \frac{6}{9-s^2} = \frac{-6}{(s+3)(s-3)}.$$

Therefore, we have

$$X(s) = \frac{-6}{(s+3)(s-3)} \quad \text{for } -3 < \text{Re}(s) < 3. \qquad \blacksquare$$



sanity check:
are stated algebraic expression and stated ROC self consistent?

yes, ROC is bounded by poles