Example 6.6. Consider the function x shown in Figure 6.5. Let $\hat{x}$ denote the Fourier transform representation of x (i.e., $\hat{x}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$, where X denotes the Fourier transform of x). Determine the values $\hat{x}(-\frac{1}{2})$ and $\hat{x}(\frac{1}{2})$.

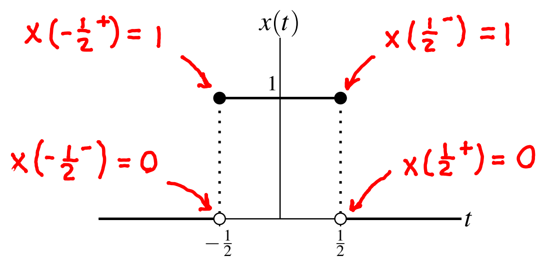

Figure 6.5: Function x.

At a point of discontinuity, the Fourier transform representation converges to the average of the left and right limits.

Solution. We begin by observing that x satisfies the Dirichlet conditions. Consequently, Theorem 6.3 applies. Thus, we have that

$$\begin{aligned}\hat{x}(-\tfrac{1}{2}) &= \tfrac{1}{2}\left[x(-\tfrac{1}{2}^-) + x(-\tfrac{1}{2}^+)\right] \\ &= \tfrac{1}{2}(0+1) \\ &= \tfrac{1}{2} \quad \text{and}\end{aligned}$$

average of left and right limits

$$\begin{aligned}\hat{x}(\tfrac{1}{2}) &= \tfrac{1}{2}\left[x(\tfrac{1}{2}^-) + x(\tfrac{1}{2}^+)\right] \\ &= \tfrac{1}{2}(1+0) \\ &= \tfrac{1}{2}.\end{aligned}$$

average of left and right limits

■