Example 5.7. The periodic square wave x in Example 5.1 has fundamental period T, fundamental frequency ω_0, and the Fourier series coefficient sequence given by

$$c_k = \begin{cases} \frac{-j2A}{\pi k} & k \text{ odd} \\ 0 & k \text{ even,} \end{cases}$$

where A is a positive constant. Find and plot the magnitude and phase spectra of x. Determine at what frequency (or frequencies) x has the most information.

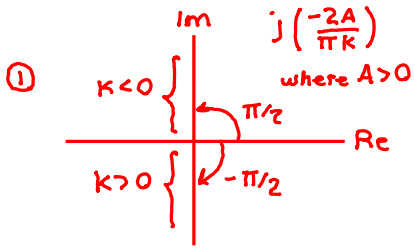
Solution. First, we compute the magnitude spectrum of x, which is given by $|c_k|$. We have

$$|c_k| = \begin{cases} \left|\frac{-j2A}{\pi k}\right| & k \text{ odd} \\ 0 & k \text{ even} \end{cases} = \begin{cases} \frac{2A}{\pi |k|} & k \text{ odd} \\ 0 & k \text{ even.} \end{cases}$$

$\left|\frac{-j2A}{\pi k}\right| = \frac{|-j2A|}{|\pi k|} = \frac{2A}{\pi |k|}$

(since $|a/b| = |a|/|b|$ and $|ab| = |a||b|$)

Next, we compute the phase spectrum of x, which is given by $\arg c_k$. Using the fact that $\arg 0 = 0$ and $\arg \frac{-j2A}{\pi k} = -\frac{\pi}{2} \operatorname{sgn} k$, we have

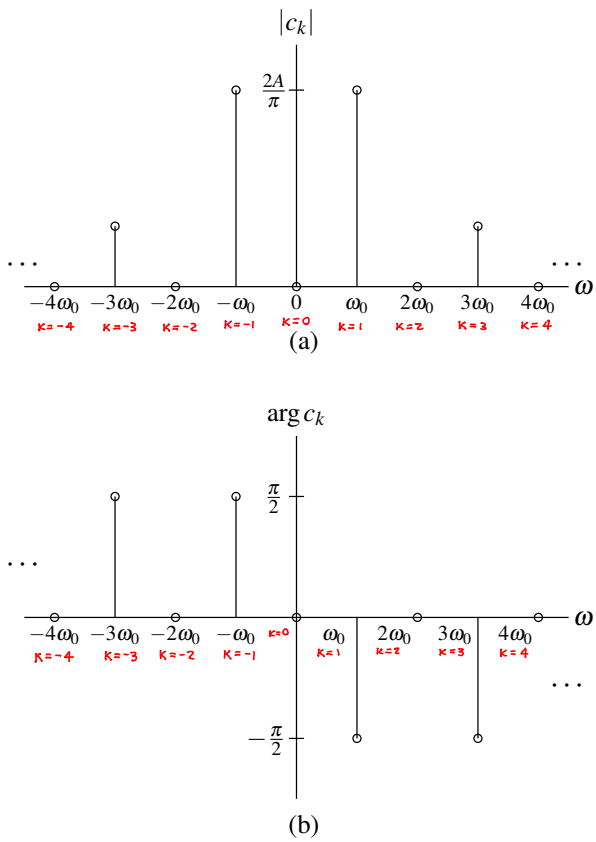


$$\arg c_k = \begin{cases} \arg \frac{-j2A}{\pi k} & k \text{ odd} \\ \arg 0 & k \text{ even} \end{cases} = \begin{cases} \frac{\pi}{2} & k \text{ odd}, k < 0 \\ -\frac{\pi}{2} & k \text{ odd}, k > 0 \\ 0 & k \text{ even} \end{cases} = \begin{cases} -\frac{\pi}{2} \operatorname{sgn} k & k \text{ odd} \\ 0 & k \text{ even.} \end{cases}$$

see ① for $\arg\left[j\left(\frac{-2A}{\pi k}\right)\right]$

$\operatorname{sgn} k = \begin{cases} 1 & k > 0 \\ 0 & k = 0 \\ -1 & k < 0 \end{cases}$

The magnitude and phase spectra of x are plotted in Figures 5.7(a) and (b), respectively. Note that the magnitude spectrum is an even function, while the phase spectrum is an odd function. This is what we should expect, since x is real. Since $|c_k|$ is largest for $k = -1$ and $k = 1$, the function x has the most information at frequencies $-\omega_0$ and ω_0. ■



magnitude spectrum

- $|c_k|$ is largest for $k=-1$ $(-\omega_0)$ and $k=1$ (ω_0)
- $|c_k|$ is even since x is real

phase spectrum

- $\arg c_k$ is odd since x is real

Figure 5.7: Frequency spectrum of the periodic square wave. (a) Magnitude spectrum and (b) phase spectrum.

