Suppose that we have a complex periodic function x with period T and Fourier series coefficient sequence c. One can easily show that the coefficient c_0 is the average value of x over a single period T. The proof is trivial. Consider the Fourier series analysis equation given by (5.2). Substituting $k = 0$ into this equation, we obtain

$$\begin{aligned} c_0 &= \left[\frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt \right] \Bigg|_{k=0} \\ &= \frac{1}{T} \int_T x(t) e^0 dt \\ &= \frac{1}{T} \int_T x(t) dt. \end{aligned}$$

from Fourier series analysis equation

evaluate at $k=0$

$e^0 = 1$

Thus, c_0 is simply the average value of x over a single period.