Example 6.17 (Frequency-domain differentiation property)**.** Find the Fourier transform X of the function

$$x(t) = t\cos(\omega_0 t),$$

where ω_0 is a nonzero real constant.

Solution. Taking the Fourier transform of both sides of the equation for x yields

$$X(\omega) = \mathcal{F}\{t\cos(\omega_0 t)\}(\omega).$$

From the frequency-domain differentiation property of the Fourier transform, we can write

$$\begin{aligned} X(\omega) &= \mathcal{F}\{t\cos(\omega_0 t)\}(\omega) \\ &= j\left(\mathcal{D}\mathcal{F}\{\cos(\omega_0 t)\}\right)(\omega), \end{aligned}$$

from definition of x

frequency-domain differentiation property

where $\mathcal{D}$ denotes the derivative operator. Evaluating the Fourier transform on the right-hand side using Table 6.2, we obtain

table of FT pairs

$$\begin{aligned} X(\omega) &= j\tfrac{d}{d\omega}\left[\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]\right] \\ &= j\pi\tfrac{d}{d\omega}\left[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right] \\ &= j\pi\tfrac{d}{d\omega}\delta(\omega-\omega_0)+j\pi\tfrac{d}{d\omega}\delta(\omega+\omega_0). \end{aligned}$$

from FT pair 1

factor out π

derivative operator is linear

■

$$\cos(w_0 t) \overset{FT}{\longleftrightarrow} \pi[\delta(w-w_0)+\delta(w+w_0)] \quad 1$$