Example 7.9 (Linearity property of the Laplace transform and pole-zero cancellation)**.** Find the Laplace transform X of the function

$$x = x_1 - x_2,$$

$$\begin{cases} x_1(t) = e^{-t}u(t) \\ x_2(t) = e^{-t}u(t) - e^{-2t}u(t) \end{cases}$$

where x_1 and x_2 are as defined in the previous example.

Solution. From the previous example, we know that

$$① \quad X_1(s) = \frac{1}{s+1} \quad \text{for } \operatorname{Re}(s) > -1 \quad \text{and}$$
$$② \quad X_2(s) = \frac{1}{(s+1)(s+2)} \quad \text{for } \operatorname{Re}(s) > -1.$$

from LT table

From the definition of X, we have

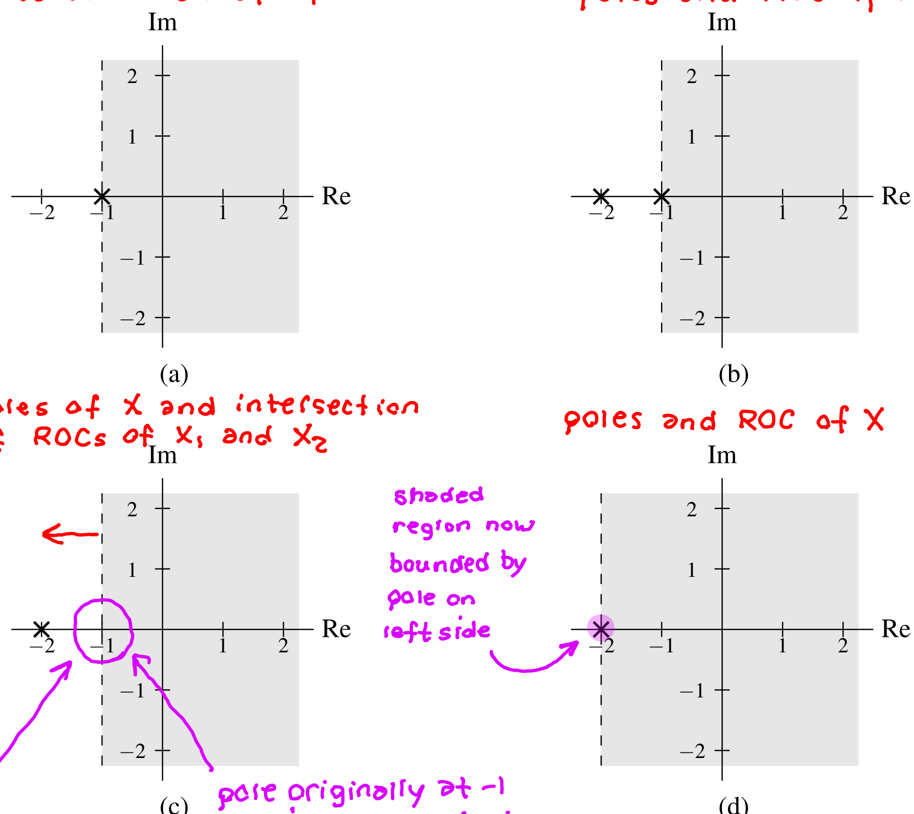
$$\begin{aligned} X(s) &= \mathcal{L}\{x_1 - x_2\}(s) && \text{linearity} \\ &= X_1(s) - X_2(s) && \text{substituting expressions for } X_1 \text{ and } X_2 \text{ in ① and ②} \\ &= \frac{1}{s+1} - \frac{1}{(s+1)(s+2)} && \text{common denominator} \\ &= \frac{s+2-1}{(s+1)(s+2)} && \text{simplify numerator} \\ &= \frac{s+1}{(s+1)(s+2)} && \text{pole-zero cancellation} \\ &= \frac{1}{s+2}. && \text{cancel common factor of } s+1 \end{aligned}$$

Now, we must determine the ROC of X. We know that the ROC of X must at least contain the intersection of the ROCs of X_1 and X_2. Therefore, the ROC must contain $\operatorname{Re}(s) > -1$. Since X is rational, we also know that the ROC must be bounded by poles or extend to infinity. Since X has only one pole and this pole is at -2, the ROC must also include $-2 < \operatorname{Re}(s) < -1$. Therefore, the ROC of X is $\operatorname{Re}(s) > -2$. In effect, the pole at -1 has been cancelled by a zero at the same location. As a result, the ROC of X is larger than the intersection of the ROCs of X_1 and X_2. The various ROCs are illustrated in Figure 7.10. So, in conclusion, we have

$$X(s) = \frac{1}{s+2} \quad \text{for } \operatorname{Re}(s) > -2. \qquad \blacksquare$$

poles and ROC of X_1

poles and ROC of X_2



poles of X and intersection of ROCs of X_1 and X_2

poles and ROC of X

can ROC be larger than intersection?

shaded region now bounded by pole on left side

ROC must be larger since shaded region not bounded by pole on left

pole originally at −1 has been cancelled

Figure 7.10: ROCs for the linearity example. The (a) ROC of X_1, (b) ROC of X_2, (c) ROC associated with the intersection of the ROCs of X_1 and X_2, and (d) ROC of X.

