Example 7.17 (Time-domain integration property)**.** Find the Laplace transform of the function

LT table

$$x(t) = \int_{-\infty}^{t} e^{-2\tau}\sin(\tau)u(\tau)d\tau.$$

Solution. From Table 7.2, we have that

$$e^{-2t}\sin(t)u(t) \overset{\text{LT}}{\longleftrightarrow} \frac{1}{(s+2)^2+1} \quad \text{for } \operatorname{Re}(s) > -2.$$

ROC is intersected with Re(s) > 0 (cannot be larger since no poles cancelled)

Using the time-domain integration property, we can deduce

multiply by 1/s

integrate

$$x(t) = \int_{-\infty}^{t} e^{-2\tau}\sin(\tau)u(\tau)d\tau \overset{\text{LT}}{\longleftrightarrow} X(s) = \underbrace{\frac{1}{s}\left(\frac{1}{(s+2)^2+1}\right)}_{\text{simplify}} \quad \text{for } \underbrace{\{\operatorname{Re}(s) > -2\} \cap \{\operatorname{Re}(s) > 0\}}_{\operatorname{Re}(s) > 0}.$$

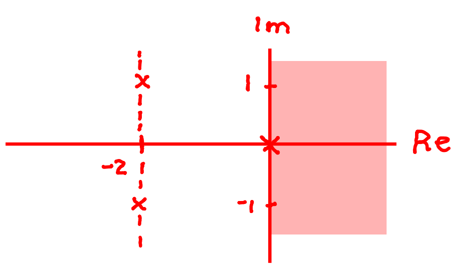
The ROC of X is $\{\operatorname{Re}(s) > -2\} \cap \{\operatorname{Re}(s) > 0\}$ (as opposed to a superset thereof), since no pole-zero cancellation takes place. Simplifying the algebraic expression for X, we have

$$X(s) = \frac{1}{s}\left(\frac{1}{(s+2)^2+1}\right) = \frac{1}{s}\left(\frac{1}{s^2+4s+4+1}\right) = \frac{1}{s}\left(\frac{1}{s^2+4s+5}\right).$$

Therefore, we have

$$X(s) = \frac{1}{s\underbrace{(s^2+4s+5)}_{(s+2-j)(s+2+j)}} \quad \text{for } \operatorname{Re}(s) > 0.$$

[Note: $s^2+4s+5 = (s+2-j)(s+2+j)$.] (s+2-j)(s+2+j) ■



sanity check:
are the stated algebraic expression and stated ROC self consistent?
yes, the ROC is bounded by poles or extends to ±∞

