Example 6.41. Let x denote a continuous-time audio signal with Fourier transform X. Suppose that $|X(\omega)| = 0$ for all $|\omega| \geq 44100\pi$. Determine the largest period T with which x can be sampled that will allow x to be exactly recovered from its samples.

44100π rad/s = 22.05kHz

Solution. The function x is bandlimited to frequencies in the range $(-\omega_m, \omega_m)$, where $\omega_m = 44100\pi$. From the sampling theorem, we know that the minimum sampling rate required is given by

from sampling theorem

$$\begin{aligned} \omega_s &= 2\omega_m \\ &= 2(44100\pi) \\ &= 88200\pi. \end{aligned}$$

$\omega_m = 44100\pi$

88200π rad/s = 44.1kHz

take reciprocal for corresponding sampling period

Thus, the largest permissible sampling period is given by

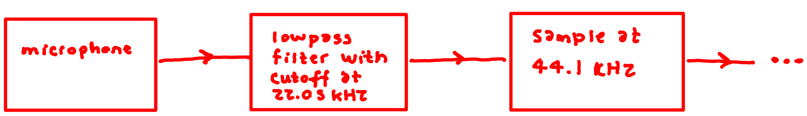
$$\begin{aligned} T &= \frac{2\pi}{\omega_s} \\ &= \frac{2\pi}{88200\pi} \\ &= \frac{1}{44100}. \end{aligned}$$

■

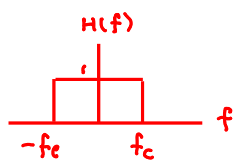
$\boxed{\text{Why does CD-quality audio use a sampling rate of 44.1 kHz?}}$

In practice, how do we ensure the audio signal to be sampled is sufficiently bandlimited?

The human auditory system (assuming pristine hearing) can sense frequencies up to about 22.05 kHz.



- filter prevents aliasing
- removed frequencies cannot be detected by humans



$f_c = 22.05$ kHz

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