

Example 3.24. Determine whether the system \mathcal{H} is invertible, where

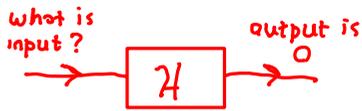
$$\mathcal{H}x(t) = \sin[x(t)].$$

Solution. Consider an input of the form $x(t) = 2\pi k$ where k is an arbitrary integer. The response $\mathcal{H}x$ to such an input is given by

$$\begin{aligned} \mathcal{H}x(t) &= \sin[x(t)] \\ &= \sin 2\pi k \\ &= 0. \end{aligned}$$

↻ ①
 ↘ substitute ①
 ↙ sin function is zero at all integer multiples of π

Thus, we have found an infinite number of distinct inputs (i.e., $x(t) = 2\pi k$ for $k = 0, \pm 1, \pm 2, \dots$) that all result in the same output. Therefore, the system is not invertible. ■



We don't know input could be $x(t) = 0$ or $x(t) = 2\pi$ or $x(t) = -2\pi$ or ... what the input is.