## Analysis of Double Side-Band Suppressed-Carrier Amplitude Modulation (DSB/SC AM)

Now, let us consider the communication system shown in Figure 6.29. This system is known as a double-side-band/suppressed-carrier (DSB/SC) amplitude modulation (AM) system. The receiver in Figure 6.29(b) contains a LTI subsystem that is labelled with its impulse response $h$. The DSB/SC AM system is very similar to the one considered earlier in Figure 6.27. In the new system, however, multiplication by a complex sinusoid has been replaced by multiplication by a real sinusoid. The new system also requires that the input signal $x$ be bandlimited to frequencies in the interval $[-\omega_b, \omega_b]$ and that

$$\omega_b < \omega_{c0} < 2\omega_c - \omega_b. \tag{6.45}$$

The reasons for this restriction will become clear after having studied this system in more detail.

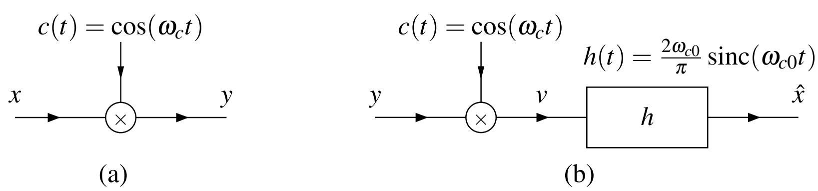


Figure 6.29: DSB/SC amplitude modulation system. (a) Transmitter and (b) receiver.

Consider the transmitter shown in Figure 6.29(a). The transmitter is a system with input $x$ and output $y$ that is characterized by the equation

$$y(t) = c(t)x(t),$$

where

$$c(t) = \cos(\omega_c t).$$

Taking the Fourier transform of both sides of the preceding equation, we obtain

$$\begin{aligned}
Y(\omega) &= \mathcal{F}\{cx\}(\omega) \\
&= \mathcal{F}\{\cos(\omega_c t)x(t)\}(\omega) && \text{Euler} \\
&= \mathcal{F}\left\{\tfrac{1}{2}[e^{j\omega_c t} + e^{-j\omega_c t}]x(t)\right\}(\omega) && \text{linearity} \\
&= \tfrac{1}{2}\left[\mathcal{F}\{e^{j\omega_c t}x(t)\}(\omega) + \mathcal{F}\{e^{-j\omega_c t}x(t)\}(\omega)\right] \\
&= \tfrac{1}{2}\left[X(\omega - \omega_c) + X(\omega + \omega_c)\right]. && \text{frequency-domain shifting property}
\end{aligned} \tag{6.46}$$

(Note that, above, we used the fact that $\cos(\omega_c t) = \frac{1}{2}(e^{j\omega_c t} + e^{-j\omega_c t})$.) Thus, the frequency spectrum of the (transmitter) output is the average of two shifted versions of the frequency spectrum of the (transmitter) input. The relationship between the frequency spectra of the input and output can be seen through Figures 6.30(a) and (d). Observe that we have managed to shift the frequency spectrum of the input signal into a different range of frequencies for transmission as desired. Next, we must determine whether the receiver can recover the original signal $x$.

Consider the receiver shown in Figure 6.29(b). The receiver is a system with input $y$ and output $\hat{x}$ that is characterized by the equations

$$v(t) = c(t)y(t) \quad = \cos(w_c t)\, y(t) \quad \text{and} \tag{6.47a}$$

$$\hat{x}(t) = v * h(t), \tag{6.47b}$$

where $c$ is as defined earlier and

$$h(t) = \frac{2\omega_{c0}}{\pi}\operatorname{sinc}(\omega_{c0} t). \tag{6.47c}$$

Let $H$, $Y$, $V$, and $\hat{X}$ denote the Fourier transforms of $h$, $y$, $v$ and $\hat{x}$, respectively. Taking the Fourier transform of $\hat{X}$ (in (6.47b)), we have

$\hat{x}(t) = v * h(t)$

$$\hat{X}(\omega) = H(\omega)V(\omega). \tag{6.48}$$

table of FT pairs

Taking the Fourier transform of $h$ (in (6.47c)) with the assistance of Table 6.2, we have

$h(t) = \frac{2w_{c0}}{\pi} \operatorname{sinc}(w_{c0} t)$

$$\begin{aligned} H(\omega) &= \mathcal{F}\left\{\tfrac{2\omega_{c0}}{\pi}\operatorname{sinc}(\omega_{c0}t)\right\}(\omega) \\ &= 2\operatorname{rect}\tfrac{\omega}{2\omega_{c0}} \\ &= \begin{cases} 2 & |\omega| \le \omega_{c0} \\ 0 & \text{otherwise.} \end{cases} \end{aligned}$$

from FT table

$\frac{B}{\pi}\operatorname{sinc}(Bt) \xleftrightarrow{\text{FT}} \operatorname{rect}\left(\frac{w}{2B}\right)$

definition of rect function

Taking the Fourier transform of $v$ (in (6.47a)) yields

$v(t) = c(t)\,y(t) = \cos(w_c t)\,y(t)$

$$\begin{aligned} V(\omega) &= \mathcal{F}\{cy\}(\omega) \\ &= \mathcal{F}\{\cos(\omega_c t)y(t)\}(\omega) \\ &= \mathcal{F}\left\{\tfrac{1}{2}\left(e^{j\omega_c t} + e^{-j\omega_c t}\right)y(t)\right\}(\omega) \\ &= \tfrac{1}{2}\left[\mathcal{F}\left\{e^{j\omega_c t}y(t)\right\}(\omega) + \mathcal{F}\left\{e^{-j\omega_c t}y(t)\right\}(\omega)\right] \\ &= \tfrac{1}{2}[Y(\omega-\omega_c) + Y(\omega+\omega_c)]. \end{aligned}$$

Euler

linearity

frequency-domain shifting property

Substituting the expression for $Y$ in (6.46) into this equation, we obtain

$Y(w) = \frac{1}{2}\left[X(w - w_c) + X(w + w_c)\right]$

$$\begin{aligned} V(\omega) &= \tfrac{1}{2}\left[\tfrac{1}{2}[X([\omega-\omega_c]-\omega_c) + X([\omega-\omega_c]+\omega_c)] + \tfrac{1}{2}[X([\omega+\omega_c]-\omega_c) + X([\omega+\omega_c]+\omega_c)]\right] \\ &= \tfrac{1}{2}X(\omega) + \tfrac{1}{4}X(\omega-2\omega_c) + \tfrac{1}{4}X(\omega+2\omega_c). \end{aligned} \tag{6.49}$$

simplify

The relationship between $V$ and $X$ can be seen via Figures 6.30(a) and (e). Substituting the above expression for $V$ into ([illegible]) and simplifying, we obtain

typo (6.48) $\leftarrow$ $\hat{X}(w) = H(w)V(w)$

$$\begin{aligned} \hat{X}(\omega) &= H(\omega)V(\omega) \\ &= H(\omega)\left[\tfrac{1}{2}X(\omega) + \tfrac{1}{4}X(\omega-2\omega_c) + \tfrac{1}{4}X(\omega+2\omega_c)\right] \\ &= \tfrac{1}{2}H(\omega)X(\omega) + \tfrac{1}{4}H(\omega)X(\omega-2\omega_c) + \tfrac{1}{4}H(\omega)X(\omega+2\omega_c) \\ &= \tfrac{1}{2}[2X(\omega)] + \tfrac{1}{4}(0) + \tfrac{1}{4}(0) \\ &= X(\omega). \end{aligned}$$

substitute $V$ from (6.49)

distribute

$\circledast$

$\operatorname{rect}\left(\frac{w}{2w_{c0}}\right) = 0$ for $|w| > w_{c0}$

$w_b < w_{c0} < 2w_c - w_b$

$\circledast$ In the above simplification, since $H(\omega) = 2\operatorname{rect}\frac{\omega}{2\omega_{c0}}$ and condition (6.45) holds, we were able to deduce that $H(\omega)X(\omega) = 2X(\omega)$, $H(\omega)X(\omega-2\omega_c) = 0$, and $H(\omega)X(\omega+2\omega_c) = 0$. The relationship between $\hat{X}$ and $X$ can be seen from Figures 6.30(a) and (f). Thus, we have that $\hat{X} = X$, implying $\hat{x} = x$. So, we have recovered the original signal $x$ at the receiver. This system has managed to shift $x$ into a different frequency range before transmission and then recover $x$ at the receiver. This is exactly what we wanted to accomplish.



The reason that this simplification is valid is probably more easily seen by looking at Figure 6.30(e).

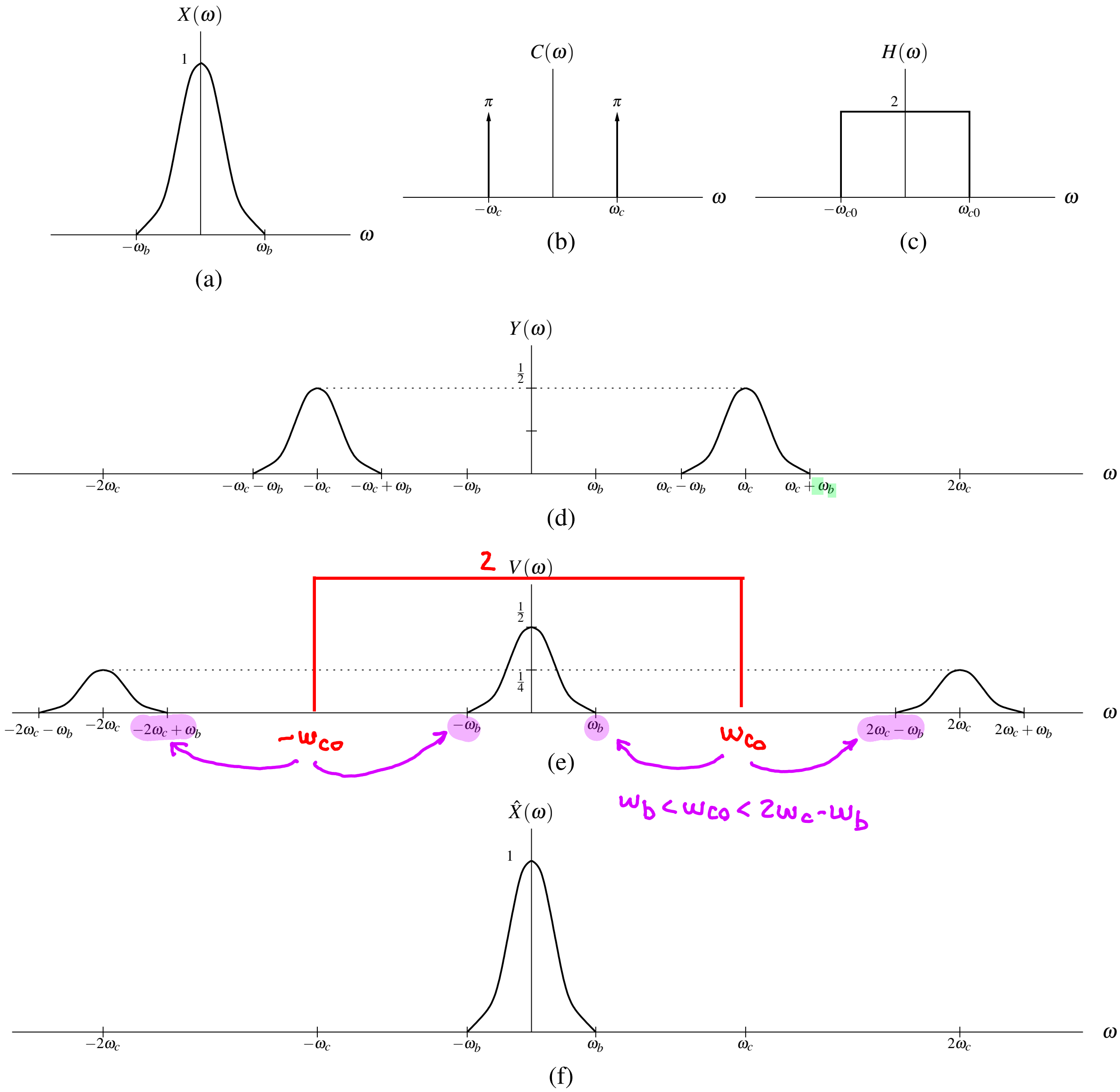


Figure 6.30: Signal spectra for DSB/SC amplitude modulation. (a) Spectrum of the transmitter input. (b) Spectrum of the sinusoidal function used in the transmitter and receiver. (c) Frequency response of the filter in the receiver. (d) Spectrum of the transmitted signal. (e) Spectrum of the multiplier output in the receiver. (f) Spectrum of the receiver output.