Answer (j).

We are asked to find the Fourier transform X of

$$x(t) = \int_{-\infty}^{5t} e^{-\tau-1} u(\tau-1) d\tau.$$

$$\boxed{\begin{aligned} x(t) &= \int_{-\infty}^{5t} e^{-\tau-1} u(\tau-1)\, d\tau \\ &= \int_{-\infty}^{5t} e^{-2} \underbrace{e^{-(\tau-1)} u(\tau-1)}_{v_1(\tau-1)} d\tau \quad \text{where } v_1(t) = e^{-t}u(t) \quad (1) \\ &= \int_{-\infty}^{5t} e^{-2} \underbrace{v_1(\tau-1)}_{v_2(\tau)} d\tau \quad \text{where } v_2(t) = v_1(t-1) \quad (2) \\ &= \underbrace{e^{-2} \int_{-\infty}^{5t} v_2(\tau)\, d\tau}_{v_3(5t)} \quad \text{where } v_3(t) = e^{-2}\int_{-\infty}^{t} v_2(\tau)\, d\tau \quad (3) \\ &= v_3(5t) \quad (4) \end{aligned}}$$

We begin by rewriting $x(t)$ as

$$(4) \rightarrow \quad x(t) = v_3(5t),$$

where

$$(1) \rightarrow \quad v_1(t) = e^{-t}u(t),$$
$$(2) \rightarrow \quad v_2(t) = v_1(t-1), \quad \text{and}$$
$$(3) \rightarrow \quad v_3(t) = \int_{-\infty}^{t} e^{-2} v_2(\tau) d\tau. = e^{-2}\int_{-\infty}^{t} v_2(\tau)\, d\tau$$

Taking the Fourier transform of both sides of each of the above equations yields

$$(5) \quad V_1(\omega) = \frac{1}{1+j\omega}, \quad \leftarrow \text{FT of (1) using FT table}$$
$$(6) \quad V_2(\omega) = e^{-j\omega} V_1(\omega), \quad \leftarrow \text{FT of (2) using time shifting property}$$
$$(7) \quad V_3(\omega) = e^{-2}\left[\frac{1}{j\omega} V_2(\omega) + \pi V_2(0)\delta(\omega)\right], \quad \text{and} \quad \leftarrow \text{FT of (3) using integration property}$$
$$(8) \quad X(\omega) = \tfrac{1}{5} V_3(\omega/5). \quad \leftarrow \text{FT of (4) using time scaling property}$$

Combining the above results, we have

$$\begin{aligned}
(8) \rightarrow \quad X(\omega) &= \tfrac{1}{5} V_3(\omega/5) \qquad \text{substitute (7)} \\
&= \tfrac{1}{5} e^{-2}\left[\left(\frac{1}{j(\omega/5)}\right) V_2(\omega/5) + \pi V_2(0)\delta(\omega/5)\right] \\
&= \tfrac{1}{5e^2}\left[\left(\frac{5}{j\omega}\right) V_2(\omega/5) + \pi V_2(0)\delta(\omega/5)\right] \qquad \text{substitute (6)} \\
&= \tfrac{1}{5e^2}\left[\left(\frac{5}{j\omega}\right) e^{-j\omega/5} V_1(\omega/5) + \pi V_1(0)\delta(\omega/5)\right] \qquad \text{substitute (5)} \\
&= \tfrac{1}{5e^2}\left[\left(\frac{5}{j\omega}\right) e^{-j\omega/5} \left(\frac{1}{1+j(\omega/5)}\right) + \pi\delta(\omega/5)\right] \\
&= \tfrac{1}{5e^2}\left[\left(\frac{5}{j\omega}\right)\left(\frac{5}{5+j\omega}\right) e^{-j\omega/5} + \pi\delta(\omega/5)\right] \qquad \text{simplify} \\
&= \tfrac{1}{5e^2}\left[\left(\frac{25}{j5\omega-\omega^2}\right) e^{-j\omega/5} + \pi\delta(\omega/5)\right].
\end{aligned}$$