Example 7.36 (Differential equation to system function)**.** A LTI system with input x and output y is characterized by the differential equation

$$y''(t) + \tfrac{D}{M} y'(t) + \tfrac{K}{M} y(t) = x(t),$$

where D, K, and M are positive real constants, and the prime symbol is used to denote derivative. Find the system function H of this system.

Solution. Taking the Laplace transform of the given differential equation, we obtain

$$s^2 Y(s) + \tfrac{D}{M} sY(s) + \tfrac{K}{M} Y(s) = X(s).$$

taking LT using time-domain differentiation property

Rearranging the terms and factoring, we have

$$\left(s^2 + \tfrac{D}{M} s + \tfrac{K}{M}\right) Y(s) = X(s).$$

rearrange terms and factor

Dividing both sides by $\left(s^2 + \tfrac{D}{M} s + \tfrac{K}{M}\right) X(s)$, we obtain

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2 + \tfrac{D}{M} s + \tfrac{K}{M}}.$$

divide both sides by $\left(s^2 + \tfrac{D}{M} s + \tfrac{K}{M}\right) X(s)$

Thus, H is given by

$$H(s) = \frac{1}{s^2 + \tfrac{D}{M} s + \tfrac{K}{M}}.$$

$Y(s) = X(s) H(s) \Rightarrow H(s) = \frac{Y(s)}{X(s)}$

■