Example 7.19. Using properties of the Laplace transform and the Laplace transform pair

$$e^{-a|t|} \overset{\text{LT}}{\longleftrightarrow} \frac{-2a}{(s+a)(s-a)} \quad \text{for } -a < \operatorname{Re}(s) < a,$$

find the Laplace transform X of the function

$$x(t) = e^{-5|3t-7|}.$$

Solution. We begin by re-expressing x in terms of the following equations:

sanity check:
$x(t) = v_2(3t)$
$= v_1(3t-7)$
$= e^{-5|3t-7|}$

$$1 \qquad v_1(t) = e^{-5|t|},$$
$$2 \qquad v_2(t) = v_1(t-7), \quad \text{and}$$
$$3 \qquad x(t) = v_2(3t).$$

In what follows, let R_{V_1}, R_{V_2}, and R_X denote the ROCs of V_1, V_2, and X, respectively. Taking the Laplace transform of the above three equations, we obtain

algebraic expression | ROC

$$4 \qquad V_1(s) = \frac{-10}{(s+5)(s-5)}, \quad R_{V_1} = (-5 < \operatorname{Re}(s) < 5),$$
← from LT of 1 using given LT pair

$$5 \qquad V_2(s) = e^{-7s}V_1(s), \quad R_{V_2} = R_{V_1},$$
← from LT of 2 using time-domain shifting property

$$6 \qquad X(s) = \tfrac{1}{3}V_2(s/3), \quad \text{and} \quad R_X = 3R_{V_2}.$$
← from LT of 3 using time-scaling property

Combining the above equations, we have

6 →
$$\begin{aligned} X(s) &= \tfrac{1}{3}V_2(s/3) \\ &= \tfrac{1}{3}e^{-7(s/3)}V_1(s/3) \\ &= \tfrac{1}{3}e^{-7s/3}V_1(s/3) \\ &= \tfrac{1}{3}e^{-7s/3}\frac{-10}{(s/3+5)(s/3-5)} \quad \text{and} \end{aligned}$$

substituting 5 for V_2

multiply

substituting 4 for V_1

6 →
$$\begin{aligned} R_X &= 3R_{V_2} \\ &= 3R_{V_1} \\ &= 3(-5 < \operatorname{Re}(s) < 5) \\ &= -15 < \operatorname{Re}(s) < 15. \end{aligned}$$

substituting 5 for R_{V2}

substituting 4 for R_{V1}

multiply

Thus, we have shown that

$$X(s) = \tfrac{1}{3}e^{-7s/3}\frac{-10}{(s/3+5)(s/3-5)} \quad \text{for } -15 < \operatorname{Re}(s) < 15. \qquad \blacksquare$$