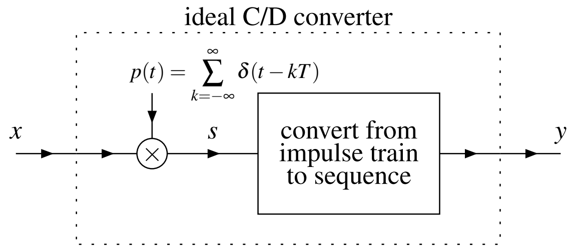
Analysis of Sampling System

s(t) = x(t) p(t)   (6.51)



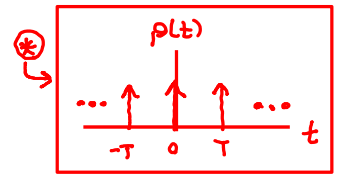


Figure 6.36: Model of ideal C/D converter with input function x and output sequence y.

Now, let us consider the above model of sampling in more detail. In particular, we would like to find the relationship between the frequency spectra of the original function x and its impulse-train sampled version s. In what follows, let X, Y, P, and S denote the Fourier transforms of x, y, p, and s, respectively. Since p is T-periodic, it can be represented in terms of a Fourier series as

from definition of Fourier series

$$p(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_s t}. \tag{6.52}$$

Using the Fourier series analysis equation, we calculate the coefficients c_k to be

Fourier series analysis equation

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$$\begin{aligned} c_k &= \tfrac{1}{T}\int_{-T/2}^{T/2} p(t)e^{-jk\omega_s t}dt \\ &= \tfrac{1}{T}\int_{-T/2}^{T/2} \delta(t)e^{-jk\omega_s t}dt \quad = \tfrac{1}{T}\int_{-\infty}^{\infty} \delta(t)\, e^{-jk\omega_s t}\, dt \\ &= \tfrac{1}{T} \\ &= \tfrac{\omega_s}{2\pi}. \end{aligned} \tag{6.53}$$

sifting property

$T = \frac{2\pi}{\omega_s}$

Substituting (6.52) and (6.53) into (6.51), we obtain

$s(t) = x(t)\,p(t)$

replace $p(t)$ by its Fourier series representation

$$\begin{aligned} s(t) &= x(t) \sum_{k=-\infty}^{\infty} \tfrac{\omega_s}{2\pi} e^{jk\omega_s t} \\ &= \tfrac{\omega_s}{2\pi} \sum_{k=-\infty}^{\infty} x(t)e^{jk\omega_s t}. \end{aligned}$$

rearrange

Taking the Fourier transform of s yields

take FT using frequency-domain shifting property

$$S(\omega) = \tfrac{\omega_s}{2\pi} \sum_{k=-\infty}^{\infty} X(\omega - k\omega_s). \tag{6.54}$$

Thus, the spectrum of the impulse-train sampled function s is a scaled sum of an infinite number of shifted copies of the spectrum of the original function x.