Example 3.9 (Sifting property example)**.** Evaluate the integral

does not have form of sifting property due to "4"

$$\int_{-\infty}^{\infty} [\sin(2\pi t)]\delta(\underbrace{4t}_{\tau}-1)dt.$$

sifting property

Solution. First, we observe that the integral to be evaluated does not quite have the same form as (3.24). So, we need to perform a change of variable. Let $\tau = 4t$ so that $t = \tau/4$ and $dt = d\tau/4$. Performing the change of variable, we obtain

change of variable

$$\begin{aligned}\int_{-\infty}^{\infty} [\sin(2\pi t)]\delta(4t-1)dt &= \int_{-\infty}^{\infty} \tfrac{1}{4}[\sin(2\pi\tau/4)]\delta(\tau-1)d\tau \\ &= \int_{-\infty}^{\infty} \left[\underbrace{\tfrac{1}{4}\sin(\pi\tau/2)}_{x(\tau)}\right]\delta(\tau-1)d\tau.\end{aligned}$$

Now the integral has the desired form, and we can use the sifting property of the unit-impulse function to write

$$\begin{aligned}\int_{-\infty}^{\infty} [\sin(2\pi t)]\delta(4t-1)dt &= \left[\tfrac{1}{4}\sin(\pi\tau/2)\right]\big|_{\tau=1} \\ &= \tfrac{1}{4}\sin(\pi/2) \\ &= \tfrac{1}{4}.\end{aligned}$$

■

Sifting property

$$\int_{-\infty}^{\infty} x(\tau)\,\delta(\tau-t_0)\,d\tau = x(t_0)$$

in this example, $x(\tau) = \frac{1}{4}\sin\left(\frac{\pi}{2}\tau\right)$ and $t_0 = 1$