Example 3.19 (Ideal integrator)**.** Determine whether the system $\mathcal{H}$ is causal, where

$$\mathcal{H}x(t) = \int_{-\infty}^{t} x(\tau)d\tau.$$

Solution. Consider the calculation of $\mathcal{H}x(t_0)$ for arbitrary t_0. We have

$$\mathcal{H}x(t_0) = \int_{-\infty}^{t_0} x(\tau)d\tau.$$

Thus, we can see that $\mathcal{H}x(t_0)$ depends only on $x(t)$ for $-\infty < t \leq t_0$. Since all of the values in this interval are less than or equal to t_0, the system is causal. ■