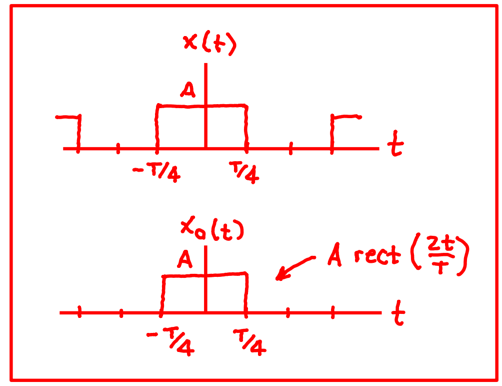

Example 6.24. Consider the periodic function x given by

$$x(t) = \sum_{k=-\infty}^{\infty} x_0(t-kT),$$

where a single period of x is given by

$$x_0(t) = A\,\mathrm{rect}\left(\tfrac{2t}{T}\right)$$

and A is a real constant. Find the Fourier transform X of the function x.

Solution. From (6.16b), we know that

$$X(\omega) = \sum_{k=-\infty}^{\infty} \omega_0 X_T(k\omega_0)\,\delta(\omega - k\omega_0)$$

$$\begin{aligned} X(\omega) &= \mathcal{F}\left\{\sum_{k=-\infty}^{\infty} x_0(t-kT)\right\}(\omega) \\ &= \sum_{k=-\infty}^{\infty} \omega_0 X_0(k\omega_0)\,\delta(\omega - k\omega_0). \end{aligned}$$

using (6.16)

table of FT pairs

So, we need to find X_0. Using the linearity property of the Fourier transform and Table 6.2, we have

$$\begin{aligned} X_0(\omega) &= \mathcal{F}\left\{A\,\mathrm{rect}\left(\tfrac{2t}{T}\right)\right\}(\omega) \\ &= A\mathcal{F}\left\{\mathrm{rect}\left(\tfrac{2t}{T}\right)\right\}(\omega) \\ &= \tfrac{AT}{2}\,\mathrm{sinc}\left(\tfrac{\omega T}{4}\right). \end{aligned}$$

from definition of x

linearity

FT table

Thus, we have that

$$\begin{aligned} X(\omega) &= \sum_{k=-\infty}^{\infty} \omega_0 \left(\tfrac{AT}{2}\right) \mathrm{sinc}\left(\tfrac{k\omega_0 T}{4}\right) \delta(\omega - k\omega_0) \\ &= \sum_{k=-\infty}^{\infty} \pi A\,\mathrm{sinc}\left(\tfrac{\pi k}{2}\right) \delta(\omega - k\omega_0). \end{aligned}$$

$\omega_0 = \frac{2\pi}{T}$

■