

Example 3.2. Let $x_1(t) = \sin(\pi t)$ and $x_2(t) = \sin t$. Determine whether the function $y = x_1 + x_2$ is periodic.

Solution. Denote the fundamental periods of x_1 and x_2 as T_1 and T_2 , respectively. We then have

$$T_1 = \frac{2\pi}{\pi} = 2 \quad \text{and} \quad T_2 = \frac{2\pi}{1} = 2\pi.$$

Here, we used the fact that the fundamental period of $\sin(\alpha t)$ is $\frac{2\pi}{|\alpha|}$. Thus, we have

$$\frac{T_1}{T_2} = \frac{2}{2\pi} = \frac{1}{\pi}.$$

Since π is an irrational number, $\frac{T_1}{T_2}$ is not rational. Therefore, y is not periodic. ■