Example 6.34 (Differential equation to frequency response)**.** A LTI system with input x and output y is characterized by the differential equation

$$7y''(t) + 11y'(t) + 13y(t) = 5x'(t) + 3x(t),$$

where x', y', and y'' denote the first derivative of x, the first derivative of y, and the second derivative of y, respectively. Find the frequency response H of this system.

Solution. Taking the Fourier transform of the given differential equation, we obtain

$$7(j\omega)^2 Y(\omega) + 11j\omega Y(\omega) + 13Y(\omega) = 5j\omega X(\omega) + 3X(\omega).$$

$\left(\frac{d}{dt}\right)^n x(t) \overset{FT}{\longleftrightarrow} (j\omega)^n X(\omega)$

move terms containing Y and X to the left- and right-hand sides, respectively and factor

Rearranging the terms and factoring, we have

$$(-7\omega^2 + 11j\omega + 13)Y(\omega) = (5j\omega + 3)X(\omega).$$

Thus, H is given by

① $\frac{Y(\omega)}{X(\omega)} = \frac{5j\omega+3}{-7\omega^2+11j\omega+13}$

divide both sides by $(-7\omega^2 + 11j\omega + 13)\,X(\omega)$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{5j\omega + 3}{-7\omega^2 + 11j\omega + 13}.$$

■

(*) ① Since system is LTI, $Y(\omega) = X(\omega)\,H(\omega) \Rightarrow H(\omega) = \frac{Y(\omega)}{X(\omega)}$