On Joint Optimization of Source and Relay Precoding Design for MIMO Downlink Channels

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***Abstract*—This paper investigates the joint precoding optimization for a relay-assisted multi-antenna downlink system. Aiming at multiuser sum capacity maximization, we first propose an iterative optimization algorithm which exploits quadratic programming approaches. Inspired by the results obtained from the iterative optimization algorithm, in order to further reduce the computational complexity, we then develop an efficient source and relay precoding strategy which diagonalizes the compound channel of the backward and the forward channel in our system. Simulation results verify the effectiveness of our proposed precoding schemes.**

I. Introduction

Relaying technology has attracted a great deal of interest due to its ability to extend the coverage in cellular networks. Recently, relay-based network architecture shows its promising potential on practical applications in future wireless systems, such as long-term evolution (LTE) and IEEE 802.16 [1]–[3]. Moreover, multiple-input multiple-output (MIMO) is a well-known technology for current radio networks to significantly improve the spectral efficiency and link reliability [4]. For these reasons, a number of studies have focused on investigating relay-assisted multi-antenna systems [5] and developing efficient precoding/relaying approaches [6], [7].

In point-to-point MIMO systems, it is known that the optimal pre-processing strategy is the singular-value decomposition (SVD)-based precoding with conventional water-filling like power allocation [4]. When a single relay is utilized between the source and the destination, however, the optimization of source precoding and relaying scheme is different and becomes more complex. Studies in [6], [7] have revealed that the optimal amplify-and-forward relaying strategy follows a similar structure as the SVD-based precoding in MIMO systems without relays. In [8], the authors further investigate the optimization problem of joint source precoding and relaying design.

On the other hand, when multiple antennas are employed at the transmitter, multiple users can be scheduled at a time for simultaneous transmission. Therefore, the multiuser MIMO downlink system is also an essential scenario in practical applications. In [5], the information theoretic limit has been investigated and dirty paper coding (DPC) is proven to be an optimal transmission strategy to achieve the full capacity of a MIMO downlink channel. Because the implementation complexity of DPC is prohibitively high for most wireless applications, a number of less-complex linear precoding technologies are then developed in [10]–[12].

Recently, an increasing attention has been paid for using relay stations in multiuser MIMO networks to deal with remote mobile users and thus to enlarge the coverage [13]–[16]. In [13], the precoding design for multiple access channels (MAC) using regenerative two-way relaying has been addressed. Concerning a single-direction relay-assisted MAC, the outage performance obtained by several simple relaying strategies is analyzed in [14]. Since it is hard to obtain the optimal precoding design for the relay-assisted multiuser MIMO systems, some suboptimal solutions have been developed in [15], [16]. More specifically, in [15], sum capacity bounds are derived for the multiuser MIMO relay system which exploits non-linear precoding at the source and linear processing at the relay. Aiming at transmit power minimization under predefined quality-of-service (QoS) requirements, an iterative joint precoding and relaying algorithm is proposed in [16].

In this paper, we consider the joint optimization of linear pre-processing at both the source and the relay in a MIMO-relay assisted multiuser downlink channel. Different from [16], we optimize the joint precoding strategy by maximizing the achievable sum capacity of our systems under fixed transmit power constraints. By using quadratic approximation, we first focus on directly solving the precoding optimization problem with efficient quadratic programming approaches. Since the original problem is non-convex, the proposed iterative method generally converges to some local optimum solutions. Numerical results still demonstrate significant performance gain. Fortunately, we also observed that the optimized precoding solutions obtained by the iterative quadratic programming method always diagonalize the compound channel of our system at high SNRs. Inspired by this observation, we then propose an efficient joint precoding and relaying scheme which diagonalizes the compound channel to several parallel single-stream channels.

II. System Model and Problem Formulation

We consider a multiuser downlink channel with an N_s-antenna base station (BS) serving L single-antenna users through an N_r-antenna relay station (RS). Since this study focuses on the problem of precoding design, like in [15], [16], we do not consider the problem of remote user scheduling in our system. We assume that only a two-hop protocol is employed by the relay station, and a half-duplex scheme is utilized. The direct links between the source and the remote users are neglected due to some facts including large path

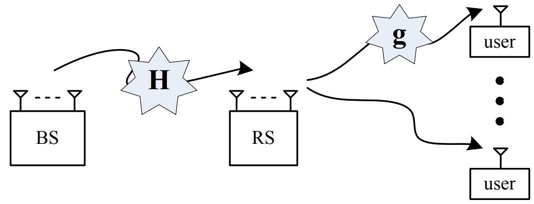

Fig. 1. System model of a relay-assisted multi-antenna downlink channel.

loss and severe shadowing effects. The system structure of the relay-assisted multi-antenna downlink channel is illustrated in Fig. 1.

Let $\mathbf{s} \in \mathbb{C}^{L\times 1}$ be the transmit symbol vector at the source. Denote $\mathbf{W}$ as the precoding matrix at the source base station (BS) before transmitting the symbols. Then, the received symbols at the relay station is

$$\mathbf{r} = \mathbf{HWs} + \mathbf{n} \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{N_r \times N_s}$ is the channel matrix from the source to the relay station, and $\mathbf{n}$ is the complex Gaussian noise with zero-mean and unit variance. The entries of the channel matrix $\mathbf{H}$ are assumed to be independent zero-mean complex Gaussian random variables of unit variance. The energy of the input symbols is normalized, i.e., $\mathbf{E}\{\mathbf{ss}^H\} = \mathbf{I}_L$. The transmit power constraint at the BS can thus be given by $\text{tr}\left(\mathbf{WW}^H\right) = P_1$, where P_1 includes the effect of path loss of the backward channel.

After receiving the symbols, the relay pre-processes the received vector by a precoding matrix $\mathbf{F}$, and then broadcast to distributed users. The transmit vector at the relay is

$$\mathbf{s}_r = \mathbf{FHWs} + \mathbf{Fn} \tag{2}$$

where the transmit power constraint at the relay station is

$$\text{tr}\left(\mathbf{FHWW}^H\mathbf{H}^H\mathbf{F}^H\right) + \text{tr}\left(\mathbf{FF}^H\right) = P_2. \tag{3}$$

Denote $\mathbf{g}_k^H$ as the $1 \times M$ channel vector between the relay station and the kth user terminal. The finally received symbol at user k is

$$y_k = \mathbf{g}_k^H\mathbf{FHWs} + \mathbf{g}_k^H\mathbf{Fn} + z_k \tag{4}$$

where z_k is the complex Gaussian noise with zero mean and unit variance. Note that the kth symbol s_k in $\mathbf{s}$ is the desired information for user k.

Representing the BS precoding matrix by $\mathbf{W} = [\mathbf{w}_1 \cdots \mathbf{w}_L]$ where $\mathbf{w}_k$ is the beamformer for data s_k to user k, we can calculate the SINR at user k as

$$\gamma_k = \frac{|\mathbf{g}_k^H\mathbf{FHw}_k|^2}{\sum_{j\neq k}|\mathbf{g}_k^H\mathbf{FHw}_j|^2 + \|\mathbf{g}_k^H\mathbf{F}\|^2 + 1}. \tag{5}$$

Thus, aiming at sum capacity maximizing, the joint precoding and relaying optimization problem can be formulated as

$$\max_{\mathbf{F},\mathbf{W}} \quad \frac{1}{2}\sum_{k=1}^{L}\log(1+\gamma_k) \tag{6a}$$

$$\text{s.t.} \quad \text{tr}\left(\mathbf{WW}^H\right) = P_1; \tag{6b}$$

$$\text{tr}\left(\mathbf{FHWW}^H\mathbf{H}^H\mathbf{F}^H\right) + \text{tr}\left(\mathbf{FF}^H\right) = P_2 \tag{6c}$$

where the factor $\frac{1}{2}$ in the objective function results from the fact that data is transmitted over two time-slots. It can be easily verified that this optimization problem is non-convex, and it is difficult to obtain the globally optimal solution. In the following sections, we will first introduce an iterative algorithm which exploits efficient quadratic programming approaches [18] to obtain a local optimal solution to this problem. Then, for implementation complexity reduction, we also develop an efficient non-iterative precoding and relaying design method using parallel transmission strategy.

III. Iterative Precoding Optimization

The proposed optimization algorithm is based on iterative mechanism, and the optimized problem in each iteration is formulated as a quadratic program which can be efficiently solved. Without loss of generality, we assume that $\mathbf{W}^{(n)}$ and $\mathbf{F}^{(n)}$ are precoding matrices obtained at the nth step. Let

$$\mathbf{W}^{(n+1)} = \mathbf{W}^{(n)} + \mathbf{\Delta}_w^{(n+1)}, \quad \mathbf{F}^{(n+1)} = \mathbf{F}^{(n)} + \mathbf{\Delta}_f^{(n+1)} \tag{7}$$

be the updated precoding matrices in the next iteration process. Then, the optimization problem in the $(n+1)$th step is to find optimal $\mathbf{\Delta}_w$ and $\mathbf{\Delta}_f$ by maximizing the objective function in (6). Accordingly, we can write the optimization problem in the $(n+1)$th step as follows:

$$\max_{\mathbf{\Delta}_f^{(n+1)},\mathbf{\Delta}_w^{(n+1)}} \quad \sum_{k=1}^{L}\left(\log\left(1+\gamma_k^{(n+1)}\right) - \log\left(1+\gamma_k^{(n)}\right)\right) \tag{8}$$

$$\begin{aligned}\text{s.t.} \quad & \text{tr}\left(\mathbf{W}^{(n+1)}\mathbf{W}^{(n+1)H}\right) = P_1; \\ & \text{tr}\left(\mathbf{F}^{(n+1)}\mathbf{HW}^{(n+1)}\mathbf{W}^{(n+1)H}\mathbf{H}^H\mathbf{F}^{(n+1)H}\right) \\ & + \text{tr}\left(\mathbf{F}^{(n+1)}\mathbf{F}^{(n+1)H}\right) = P_2.\end{aligned}$$

In order to make the above optimization problem more tractable, the challenge here is to reformulate the above problem into a standard quadratic program. Before doing this, we need an assumption that both matrices $\mathbf{\Delta}_w$ and $\mathbf{\Delta}_f$ are element-wise very small. Note that this assumption is critical for our proposed iterative algorithm and will definitely be satisfied because we will add it as an additional constraint in the above problem. Under this assumption, we then implement quadratic approximation to the objective function in (8) and apply linear approximation to the constraints. Following detailed manipulations provided in Appendix A, we reformulate the problem in (8) into a standard quadratic program:

$$\min_{\mathbf{x}} \quad \frac{1}{2}\mathbf{x}^T\left(\sum_{k=1}^{L}\frac{\mathbf{p}_k\mathbf{p}_k^T}{\left(1+\gamma_k^{(n)}\right)^2}\right)\mathbf{x} - \left(\sum_{k=1}^{L}\frac{\mathbf{p}_k^T}{1+\gamma_k^{(n)}}\right)\mathbf{x} \tag{9}$$

$$\begin{aligned}\text{s.t.} \quad & 2\begin{bmatrix}\mathbf{d}^T \\ \mathbf{e}^T\end{bmatrix}\mathbf{x} = \begin{bmatrix}\hat{p}_1 \\ \hat{p}_2\end{bmatrix}; \\ & |x_i| \le \tau; \quad i = \{1,2,\cdots,2(N_sL+N_r^2)\},\end{aligned}$$

where $\mathbf{p}_k$ is given by (21), and $\mathbf{d}$, $\mathbf{e}$, $\hat{p}_1$, and $\hat{p}_2$ are from (22), (23), (24), and (25), respectively. The desired vector $\mathbf{x}$ defined in (18) is made up of the elements of optimized $\mathbf{\Delta}_w$ and $\mathbf{\Delta}_f$. Note that the bound constraints with a small predetermined τ are used to guarantee the assumption of a small $\mathbf{x}$, i.e., small precoding update steps. Note that this problem (9) can be efficiently solved [18], and, for instance, by directly using `quadprog` in MATLAB. At this point, we are now ready to present the detailed description of our proposed iterative joint precoding optimization method in Algorithm 1.

Algorithm 1 : Iterative Joint Precoding Optimization

1: Select an initial point, i.e., $\mathbf{W}^{(0)}$ and $\mathbf{F}^{(0)}$.
2: The nth iteration process:
Given $\mathbf{W}^{(n)}$ and $\mathbf{F}^{(n)}$, find the optimal solution of $\mathbf{x}$ by solving problem (9). Then, obtain $\Delta_w^{(n+1)}$ and $\Delta_f^{(n+1)}$ by reshaping $\mathbf{x}$ according to (18).
3: Find α^*, a value of $\alpha \in [0,\ 1]$ that maximizes the objective function in (6) with $\mathbf{W} = \mathbf{W}^{(n)} + \alpha\mathbf{\Delta}_w^{(n+1)}$ and $\mathbf{F} = \mathbf{F}^{(n)} + \alpha\mathbf{\Delta}_f^{(n+1)}$, using a line search.
4: Update $\mathbf{W}^{(n+1)} = \mathbf{W}^{(n)} + \alpha^*\mathbf{\Delta}_w^{(n+1)}$ and $\mathbf{F}^{(n+1)} = \mathbf{F}^{(n)} + \alpha^*\mathbf{\Delta}_f^{(n+1)}$.
5: End the nth iteration process. Go back to *Step* 2 until $\|\alpha^*\mathbf{x}\|^2 \leq \epsilon$ converges for a very small ϵ.
6: Scale the obtained solution of $\mathbf{W}$ and $\mathbf{F}$ according to the power constraints in (6b) and (6c), respectively.

Here, we provide some remarks on the proposed iterative optimization algorithm:

- Since the original problem in (6) is non-convex, it is generally difficult to obtain the globally optimal solution to this problem. Although this implies that the proposed iterative optimization algorithm converges to some local optimum solutions, numerical results still demonstrate significant performance gain.
- Line search has been popular and essential to many optimization algorithms in that an iteration step is not considered complete until a line search step is carried out [18]. In *Step* 3, we introduce a line search step for finding the optimal scalar α^* before updating the required precoding matrices. This scalar is utilized to guarantee the convergence of our iterative algorithm.
- We apply linear approximation to obtain the power constraints in (9). Although we restrict that $\mathbf{x}$ is very small, the original power constraints in (6) are only satisfied within an acceptable tolerance. In order to strictly guarantee the original transmit power constraints and also for fair comparison, we add the last *Step* 6.
- The final performance of our iterative algorithm also depends on the selected initial point. Fortunately, using different initial points, we observed that the obtained precoding matrices always diagonalize the compound channel of our system at high SNRs. This observation inspired us to further develop an efficient precoding design strategy in the following section. It is an alternative to balance the computational complexity and the achievable sum capacity performance.

IV. Efficient Parallel Transmission Scheme

In this section, we propose an efficient source and relay precoding design strategy by diagonalizing the compound channel of our system, i.e., $\mathbf{GFHW}$, where $\mathbf{G} = [\mathbf{g}_1 \cdots \mathbf{g}_L]^H$ represents the concatenation of relay-to-user channels. This parallel transmission scheme exploits SVD-based precoding at the source, and combines the SVD-based receiving matrix and the zeroforcing beamforming technique at the relay station. Note that the precoding matrices designed for parallel transmissions follows a similar structure of the channel inversion based precoding design in [16]. However, we modified the design of precoding matrices in order to make it more suitable for some practical applications without channel information feedback from the relay. The differences mainly lie in two aspects. First, the source precoder design in [16] requires both channel information of $\mathbf{H}$ and $\mathbf{G}$, while our proposed source precoder only depends on $\mathbf{H}$. Moreover, we design the precoding by aiming at sum capacity maximization which allows an easy way to find the optimal eigenmode-matching for source-to-relay and relay-to-user channels.

Using SVD, the source-to-relay channel can be decomposed by

$$\mathbf{H} = \mathbf{U\Sigma V}^H \tag{10}$$

where $\mathbf{\Sigma}$ is the diagonal matrix with singular-values sorted in decreasing order. Let $\overline{\mathbf{V}} \in \mathbb{C}^{N_s \times L}$ and $\overline{\mathbf{U}} \in \mathbb{C}^{N_r \times L}$ represent the first L columns of $\mathbf{V}$ and $\mathbf{U}$, respectively. Denote $\overline{\mathbf{\Sigma}}$ as the primary $L \times L$ diagonal block of $\mathbf{\Sigma}$. Then, the proposed structure of the precoding matrices are

$$\mathbf{W} = \sqrt{\frac{P_1}{L}}\overline{\mathbf{V}}\mathbf{\Pi} \tag{11}$$

and

$$\mathbf{F} = \sqrt{\rho}\widehat{\mathbf{F}}\mathbf{\Pi}\overline{\mathbf{U}}^H \tag{12}$$

where $\widehat{\mathbf{F}} = [\mathbf{f}_1 \cdots \mathbf{f}_L]$ with $\mathbf{f}_k$ for user k being the normalized kth column of the matrix $\mathbf{G}^H\left(\mathbf{G}\mathbf{G}^H\right)^{-1}$, $\mathbf{\Pi}$ is a permutation matrix, and

$$\rho = \frac{P_2}{\operatorname{tr}\left(\widehat{\mathbf{F}}^H\widehat{\mathbf{F}}\left(\frac{P_1}{L}\overline{\mathbf{\Sigma}}_\pi\overline{\mathbf{\Sigma}}_\pi^H + \mathbf{I}\right)\right)} \tag{13}$$

with $\overline{\mathbf{\Sigma}}_\pi = \mathbf{\Pi}\overline{\mathbf{\Sigma}}\mathbf{\Pi}$ is a scalar to make the power constraint in (6c) satisfied.

After determining the proposed structure of precoder matrices, we need to choose an optimized $\mathbf{\Pi}$ by maximizing the sum capacity of our system. By substituting the precoders (11) and (12) into (5), the achievable SINR can be calculated by

$$\gamma_k^{\text{paral}} = \frac{\sigma_{\pi(k)}^2 P_1}{L}\left(1 - \frac{1}{\rho|\mathbf{g}_k^H\mathbf{f}_k|^2 + 1}\right) \tag{14}$$

where $\sigma_{\pi(k)}$ is the kth diagonal element of $\mathbf{\Pi}\overline{\mathbf{\Sigma}}\mathbf{\Pi}$, that is, $\pi(k)$th diagonal element of $\overline{\mathbf{\Sigma}}$. Accordingly, the achievable capacity is given by

$$\mathcal{R} = \frac{1}{2}\sum_{k=1}^{L}\log\left(1 + \frac{\sigma_{\pi(k)}^2 P_1}{M}\left(1 - \frac{1}{\rho|\mathbf{g}_k^H\mathbf{f}_k|^2 + 1}\right)\right). \tag{15}$$

Thus far, its easy to find the optimal $\mathbf{\Pi}$ by maximizing the sum capacity $\mathcal{R}$. Given the values of σ_k^2 and $|\mathbf{g}_k^H \mathbf{f}_k|^2$, we generally need to compare $\mathcal{R}$ obtained by using all $L!$ possible permutation patterns, and then pick out the best one.

V. Simulation Results

This section presents simulation results of our proposed precoding optimization strategies. We average the sum capacity over 2000 random channel realizations. We test the system with $P_1 = P_2$ which indicates that the backward and the forward channels have the same average channel energy including the transmit power constraints and the effects of path loss. For comparison, we implement four different precoding strategies as follows:

- Identity-based precoding: $\mathbf{W} = \eta \widehat{\mathbf{W}}$ where $\widehat{\mathbf{W}}$ is an $N_s \times L$ matrix with 1 as its diagonal elements and zeros elsewhere. $\mathbf{F} = \mu \mathbf{I}_{N_r}$. η and μ are scalars to make the power constraint in (6b) and (6c) satisfied.
- Match filter based precoding: $\mathbf{W} = \frac{P_1}{L}\overline{\mathbf{V}}$, and $\mathbf{F} = \mu \widetilde{\mathbf{F}} \overline{\mathbf{U}}^H$ where $\widetilde{\mathbf{F}} = [\mathbf{g}_1/\|\mathbf{g}_1\| \ \cdots \ \mathbf{g}_L/\|\mathbf{g}_L\|]$ and μ is a scalar to guarantee the power constraint in (6c).
- Parallel transmission with eigenmode matching: the efficient precoding design proposed in Section IV.
- Iterative joint optimization proposed in Algorithm 1. We implement the iterative joint precoding algorithm with $\tau = 0.03$ [1] and $\epsilon = 10^{-5}$. We use the closed-form precoding design in Section IV with permutation matrix $\mathbf{\Pi} = \mathbf{I}$ as the initial point for our iterative optimization algorithm.

Fig. 2 compares the ergodic sum capacity obtained by the above four different precoding strategies under $N_s = N_r = L = 2$ as a function of channel average SNRs. From the results, we find that significant performance gain is achieved by using properly designed precoding methods. The iterative algorithm achieves the maximum sum capacity at the cost of relatively high computational complexity. Its achievable sum capacity can serve as an upper bound for that obtained by the other efficiently-designed precoding approaches. Fig. 3 depicts the sum capacity comparison under $N_s = N_r = L = 4$. Similar observations can be concluded from the results in Fig. 3. By comparing the two figures, we find that the iterative algorithm provides more performance gain for systems equipped with more antennas and serving more mobile users.

Among the closed-form precoding design methods, it can be found that the match filter based precoding design outperforms the others from small to moderate SNRs. When SNR grows large, however, the parallel transmission scheme achieves better performance than the match filter based precoding. This coincides with our observation from the results obtained by using iterative joint optimization algorithm. At high SNR regimes, parallel transmission structure seems to be the best

[1]By defining $\rho_1 = \frac{P_1}{L}$ and ρ_2 from (13), we use Algorithm 1 to find the optimized solutions to $\mathbf{W}$ and $\mathbf{F}$ which have been normalized by $\sqrt{\rho_1}$ and $\sqrt{\rho_2}$, respectively. This allows us to use a same τ in the proposed iterative algorithm under different SNRs configurations.

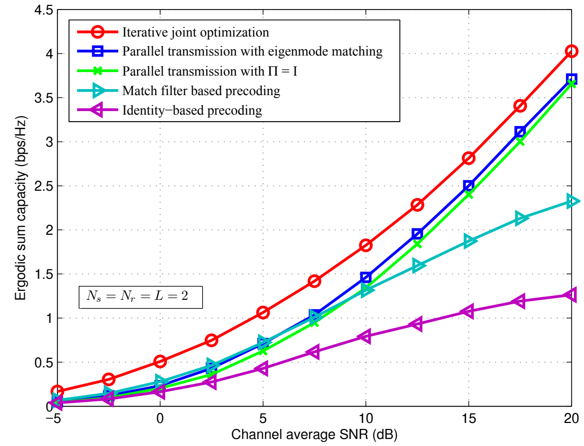

Fig. 2. Comparison of ergodic sum capacity achieved by different precoding schemes as a function of $P_1 = P_2$ = SNR in dB under $N_s = N_r = L = 2$.

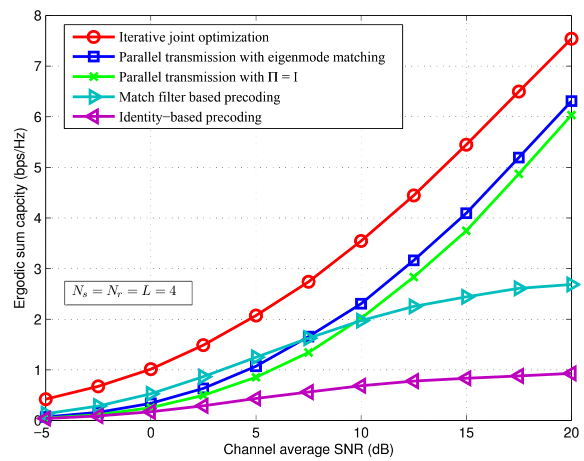

Fig. 3. Comparison of ergodic sum capacity achieved by different precoding schemes as a function of $P_1 = P_2$ = SNR in dB under $N_s = N_r = L = 4$.

precoding strategy. This is because the sum capacity performance is dominated by the multi-user interference when SNR becomes large.

VI. Conclusion

We propose an iterative algorithm for jointly optimizing the precoding at both the source and the relay in a multiuser multi-antenna downlink channel. The iterative optimization algorithm shows that the obtained precoding matrices always diagonalize the compound channel of our system at high SNRs. Based on this observation, we then present an efficient precoder design strategy which balances the computational complexity and the achievable sum capacity for our system. Empirical results compare the performance achieved by different precoding strategies. It shows that the iterative optimization algorithm provides the best performance with the cost of much more computational complexity. Efficient parallel transmission precoding strategy is an alternative which provides a different tradeoff between the performance and the implementation complexity.

APPENDIX A
QUADRATIC PROGRAM REFORMULATION OF (8)

Due to space constraints, we only provide an outline of the reformulation process. The complete description will be presented in an extended journal version of this paper. Start with the following definition

$$x_k = |\mathbf{g}_k^H \mathbf{F}\mathbf{H}\mathbf{w}_k|^2. \quad (16)$$

By substituting (7) into the above expression and separating the real and imaginary parts in the variables, we can re-write the updated $x_k^{(n+1)}$ using linear approximation by

$$x_k^{(n+1)} \approx x_k^{(n)} + 2\mathbf{a}_k^T \mathbf{x} \quad (17)$$

where

$$\mathbf{x} = [\text{vec}(\mathbf{\Delta}_w)_R^T, \text{vec}(\mathbf{\Delta}_w)_I^T, \text{vec}(\mathbf{\Delta}_f)_R^T, \text{vec}(\mathbf{\Delta}_f)_I^T]^T \quad (18)$$

with vec$(\cdot)$ being the vectorization of a matrix, and subscripts R and I representing the real and imaginary parts, respectively.

$$\mathbf{a}_k^T = \left[\mathbf{0}_{1\times N_s(k-1)}, \mathbf{u}_{k,k,R}^T, \mathbf{0}_{1\times N_s(L-1)}, \mathbf{u}_{k,k,I}^T, \mathbf{0}, \mathbf{v}_{k,k,R}^T, \mathbf{v}_{k,k,I}^T\right]$$

where $\mathbf{u}_{k,j} = \text{vec}\left(\mathbf{H}^H \mathbf{F}^{(n)H} \mathbf{g}_k \mathbf{g}_k^H \mathbf{F}^{(n)} \mathbf{H} \mathbf{w}_j^{(n)}\right)$ and $\mathbf{v}_{k,j} = \text{vec}\left(\mathbf{g}_k \mathbf{g}_k^H \mathbf{F}^{(n)} \mathbf{H} \mathbf{w}_j^{(n)} \mathbf{w}_j^{(n)H} \mathbf{H}^H\right)$. Similarly, by defining the denominator of γ_k in (5) by y_k, we obtain the linear approximation of y_k as

$$y_k^{(n+1)} \approx y_k^{(n)} + 2\mathbf{b}_k^T \mathbf{x} \quad (19)$$

where

$$\mathbf{b}_k^T = \left[\mathbf{c}_{k,j,R}^T, \mathbf{c}_{k,j,I}^T, \left(\mathbf{t}_k^T + \sum_{j\neq k} \mathbf{v}_{k,j}^T\right)_R, \left(\mathbf{t}_k^T + \sum_{j\neq k} \mathbf{v}_{k,j}^T\right)_I\right]$$

with $\mathbf{c}_{k,j}^T = [\mathbf{u}_{k,1}^T, \cdots, \mathbf{u}_{k,(k-1)}^T, \mathbf{0}_{1\times N_s}, \mathbf{u}_{k,(k+1)}^T, \cdots, \mathbf{u}_{k,L}^T]$ and $\mathbf{t}_k = \text{vec}\left(\mathbf{g}_k \mathbf{g}_k^H \mathbf{F}^{(n)}\right)$. Further by applying Taylor expansion to $\gamma_k = x_k / y_k$ with respect to x_k and y_k, from (17) and (19), we have

$$\gamma^{(n+1)} \approx \gamma^{(n)} + \left(\frac{2}{y^{(n)}}\mathbf{a}_k - \frac{2x^{(n)}}{y^{(n)2}}\mathbf{b}_k\right)^T \mathbf{x}. \quad (20)$$

By defining

$$\mathbf{p}_k = \frac{2}{y^{(n)}}\mathbf{a}_k - \frac{2x^{(n)}}{y^{(n)2}}\mathbf{b}_k, \quad (21)$$

and applying second-order Taylor expansion to the objective function in (6) with respect to γ in (20), we obtain the quadratic-form objective in (9).

Similarly, by implementing linear approximation to the power constraint in (6), we finally have the quadratic program in (9), where

$$\mathbf{d}^T = \left[\text{vec}\left(\mathbf{W}^{(n)}\right)_R^T, \text{vec}\left(\mathbf{W}^{(n)}\right)_I^T, \mathbf{0}_{1\times 2N_r^2}\right], \quad (22)$$

$$\begin{aligned}\mathbf{e}^T = \Big[&\text{vec}\left(\mathbf{H}^H \mathbf{F}^{(n)H} \mathbf{F}^{(n)} \mathbf{H} \mathbf{W}^{(n)}\right)_R^T, \\ &\text{vec}\left(\mathbf{H}^H \mathbf{F}^{(n)H} \mathbf{F}^{(n)} \mathbf{H} \mathbf{W}^{(n)}\right)_I^T, \\ &\text{vec}\left(\mathbf{F}^{(n)} \mathbf{H} \mathbf{W}^{(n)} \mathbf{W}^{(n)H} \mathbf{H}^H + \mathbf{F}^{(n)}\right)_R^T, \\ &\text{vec}\left(\mathbf{F}^{(n)} \mathbf{H} \mathbf{W}^{(n)} \mathbf{W}^{(n)H} \mathbf{H}^H + \mathbf{F}^{(n)}\right)_I^T\Big], \end{aligned} \quad (23)$$

$$\hat{p}_1 = P_1 - \text{tr}\left(\mathbf{W}^{(n)} \mathbf{W}^{(n)H}\right), \quad (24)$$

and

$$\hat{p}_2 = P_2 - \text{tr}\left(\mathbf{F}^{(n)}\left(\mathbf{H}\mathbf{W}^{(n)}\mathbf{W}^{(n)H}\mathbf{H}^H + \mathbf{I}\right)\mathbf{F}^{(n)H}\right). \quad (25)$$

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