

**FULL-WAVE ANALYSIS OF MULTIPLE LOSSY MICROSTRIP LINES ON MULTILAYERED BI-ANISOTROPIC SUBSTRATES AND IMPERFECT GROUND METALLIZATION**

Zhenglian Cai and Jens Bornemann

Laboratory for Lightwave Electronics, Microwaves and Communications (LLiMiC)  
 Department of Electrical and Computer Engineering  
 University of Victoria, Victoria B.C. Canada V8W 3P6

**ABSTRACT**

An extended spectral-domain immittance approach is used to analyze multiconductor systems on layered bi-anisotropic substrates. Apart from the losses of the bi-anisotropic layers, the method takes into account the metallization thickness and losses of both the ground metallization and the multi-conductor system which can consist of conventional or high-Tc superconductors. It is demonstrated that due to the order, permittivity and anisotropy of substrate layers, the propagation characteristics of a multi-conductor system can be influenced to obtain a frequency range of almost identical complex propagation constants for all fundamental modes involved. Such a behavior can be advantageously utilized in future applications of high-speed interconnects and high-directivity couplers. Since the algorithm is CPU-time efficient and operational on modern workstations, the developed model offers an attractive solution for modern MMIC design purposes. At the example of a coupled pair of microstrip lines on anisotropic substrate, results are compared with previously published data and are found to be in excellent agreement.

**I. INTRODUCTION**

Multi-conductor microstrip lines have become increasingly important in high-speed digital integrated circuits applications [1, 2]. The design of such components, however, suffers from problems associated with a significant anisotropic behavior of the substrates when operated at high frequencies [3]. The design task is even more complicated by the introduction of low-loss, low-dispersion high-Tc superconductors which will offer attractive solutions to current problems. Investigations considering the influences of some of the material parameters related to the above applications have been carried out in recent years, particularly on the characteristics of multiple coupled microstrip lines on either single or multilayered substrates with lossy and anisotropic medium. Although some of the analysis techniques used for multiple conductor structures are based on full-wave methods, mostly isotropic media are considered [1, 2, 4]. Coupled microstrip lines on anisotropic substrate have been investigated in [3, 5-7], but no attention has been paid to multiple lossy conductors as well as to the calculation of the characteristic impedance.

Therefore, the purpose of this work is to present a technique for the analysis of multiple lossy microstrip lines on complex, multilayered substrates of dielectric and/or magnetic anisotropy. Due to the flexibility of the analysis, efforts are not only focused on propagation constant and characteristic impedance calculations but also on the computation of losses caused by conventional or high-Tc superconductors, anisotropic substrates and imperfect ground metallization.

**II. THEORY**

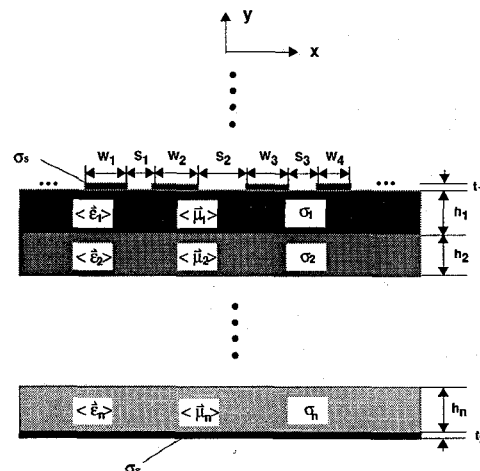
The geometry of multiple lossy microstrip lines on multilayered bi-anisotropic media and imperfect ground metallization is shown in Fig.1. The concept of the extended spectral-domain immittance approach (SDIA) as proposed in [8-10] is applied, which is very suitable for the analysis of multilayered anisotropic structures. The substrates shown in Fig. 1 are modelled by utilizing bi-anisotropic tensor properties, which are expressed as

$$\langle \vec{\epsilon} \rangle = \epsilon_o \begin{bmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \quad \langle \vec{\mu} \rangle = \mu_o \begin{bmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{bmatrix} \quad (1)$$

where  $\epsilon_x, \epsilon_y, \epsilon_z, \mu_x, \mu_y, \mu_z$  are complex quantities to account for the losses in the materials.

In order to find the dyadic Green's function of the structure, the process commences with the decomposition of the electromagnetic field into TM-to-y and TE-to-y waves by introducing coordinate transforms [10]. The transverse propagation constants in y direction can be obtained from Maxwell's equations

$$\gamma_m^2 = \left( \frac{\epsilon_x}{\epsilon_y} \alpha^2 + \frac{\epsilon_z}{\epsilon_y} \beta^2 \right) - \frac{\omega^2 \mu_o \epsilon_o \mu_x \mu_z (\epsilon_x \alpha^2 + \epsilon_z \beta^2)}{\mu_x \alpha^2 + \mu_z \beta^2} \quad (2)$$



**Fig. 1** Geometry of lossy multiconductor transmission lines on bi-anisotropic substrates and imperfect ground metallization.

OF2

$$\gamma_e^2 = \left( \frac{\mu_x}{\mu_y} \alpha^2 + \frac{\mu_z}{\mu_y} \beta^2 \right) - \frac{\omega^2 \mu_o \epsilon_o \epsilon_x \epsilon_z (\mu_x \alpha^2 + \mu_z \beta^2)}{\epsilon_x \alpha^2 + \epsilon_z \beta^2} \quad (3)$$

where  $\alpha$  and  $\beta$  are the propagation constants in  $x$  and  $z$  direction, respectively. The wave admittances associated with (2), (3) can be obtained as

$$Y_{TM} = \frac{j\omega \epsilon_o \epsilon_x \epsilon_z (\mu_x \alpha^2 + \mu_z \beta^2)}{\gamma_m (\mu_x \epsilon_z \alpha^2 + \mu_z \epsilon_x \beta^2)} \quad (4)$$

and

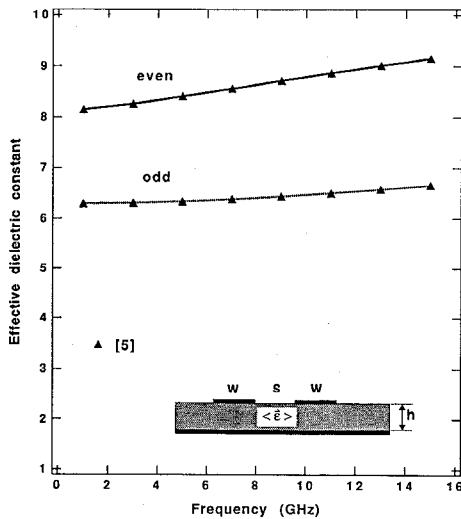
$$Y_{TE} = \frac{\gamma_e (\mu_z \epsilon_x \alpha^2 + \mu_x \epsilon_z \beta^2)}{j\omega \epsilon_o \mu_x \mu_z (\epsilon_x \alpha^2 + \epsilon_z \beta^2)} \quad (5)$$

After obtaining the impedance dyadic Green's function by applying SDIA, the function is modified by considering a complex boundary condition to incorporate the finite thickness of conductors as well as the lossy ground metallization, which can be either conventional conductors or superconductors [9]. The tangential current density is expressed in terms of appropriate basis functions

$$J_{zi}(x) = P_i \frac{\cos \{ (i-1) \pi (x-b_i) / 2w_i + (i-1) (\pi/2) \}}{\sqrt{w_i - (x-b_i)^2}} \quad (6)$$

$$J_{xi}(x) = P_i \frac{\sin \{ i\pi (x-b_i) / w_i + i\pi \}}{\sqrt{w_i - (x-b_i)^2}} \quad (7)$$

where  $w_i$  is the width of strip  $i$ , and  $b_i$  is the center coordinator of the strip. Different mode configurations can be selected by specifying -1 or +1 for  $P_i$ . The characteristic impedance is calculated using the power-current definition.



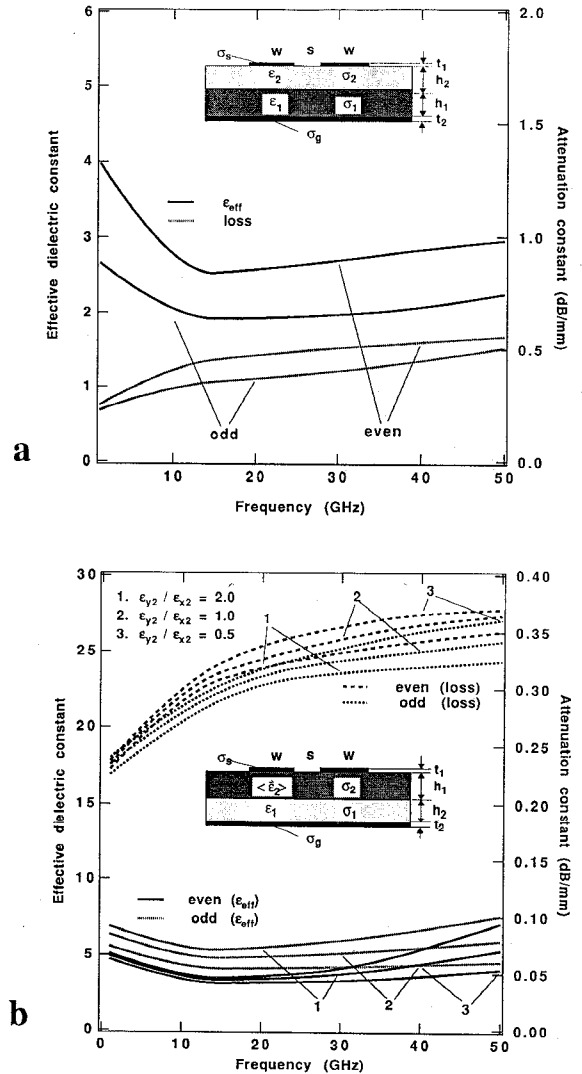
**Fig. 2** Comparison between this theory and [5] at the example of the effective dielectric constants of coupled microstrip lines;  $w=0.6\text{mm}$ ,  $s=0.4\text{mm}$ ,  $h=0.635\text{mm}$ ,  $\epsilon_x=\epsilon_z=9.4$ ,  $\epsilon_y=11.6$ .

### III. NUMERICAL RESULTS

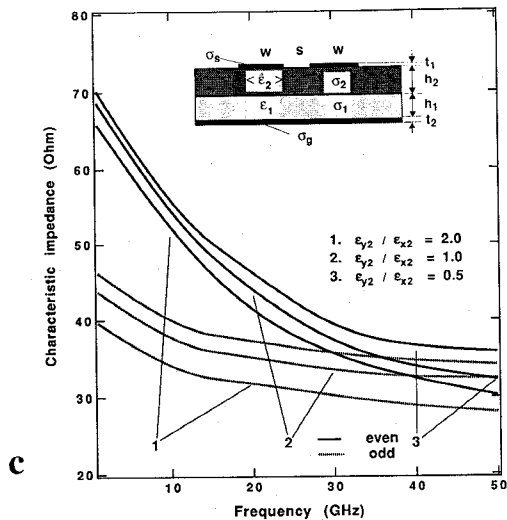
To verify this theory, a coupled pair of microstrip lines on dielectrically anisotropic sapphire substrate ( $\epsilon_x=\epsilon_z=9.4$ ,  $\epsilon_y=11.6$ ) is analyzed and compared with data presented in [5] where ideal and infinitely thin conductors and ground metallizations are assumed. As is demonstrated in Fig. 2, excellent agreement for both fundamental modes is obtained. The structure shows the typical characteristic known in MIC's:  $\epsilon_{\text{eff}}(\text{even}) > \epsilon_{\text{eff}}(\text{odd})$ , and both curves increase steadily with frequency.

As soon as (conventional or high-Tc super-) conductor losses as well as dielectric losses are included, however, the behavior changes to display the negative slope at lower frequencies (Fig. 3a) which is known from slow-wave applications, e.g. [1]. Note that the general tendency of  $\epsilon_{\text{eff}}(\text{even}) > \epsilon_{\text{eff}}(\text{odd})$  is preserved and that the attenuation constant of the even mode is higher than that of the odd mode as expected from the even- and odd-mode field distributions.

This behavior is maintained as long as the bottom substrate has the generally higher dielectric constant of the two layers (Fig. 3a). However, if the upper layer has higher permittivity



**b**

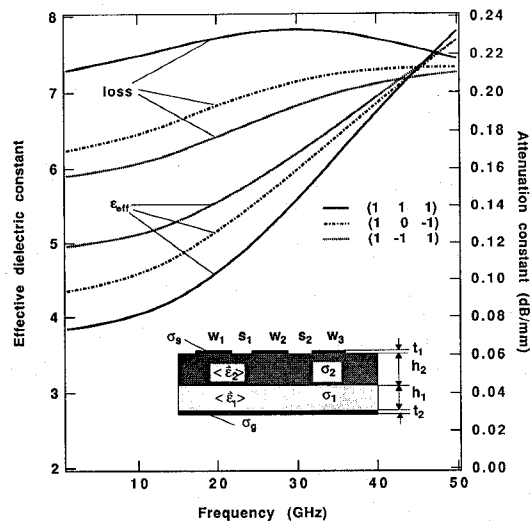


**Fig. 3** Effective dielectric constant, attenuation constant and characteristic impedance of coupled high-Tc superconducting microstrip lines;

- a)  $\epsilon_{x1}=\epsilon_{y1}=\epsilon_{z1}=9.4$ ,  $\epsilon_{x2}=\epsilon_{y2}=\epsilon_{z2}=2.4$ ,  $\sigma_1=0.3\text{S/m}$ ,  $\sigma_2=0.5\text{S/m}$ ,  $w=0.6\text{mm}$ ,  $s=0.5\text{mm}$ ,  $h_1=0.6\text{mm}$ ,  $h_2=0.3\text{mm}$ ; superconductor:  $t_1=0.2\mu\text{m}$ ,  $\sigma_s=200\text{S/mm}$ ,  $T/T_c=0.5$ ,  $\lambda_{\text{eff}}=3000\text{\AA}$ ; ground plane:  $t_2=0.5\text{mm}$ ,  $\sigma_g=40\text{S}/\mu\text{m}$ ;
- b)  $\epsilon_{x1}=\epsilon_{y1}=\epsilon_{z1}=2.4$ ,  $\epsilon_{x2}=\epsilon_{z2}=9.4$ ,  $h_1=0.3\text{mm}$ ,  $h_2=0.6\text{mm}$ , other parameters according to a);
- c) characteristic impedance for the structure of b).

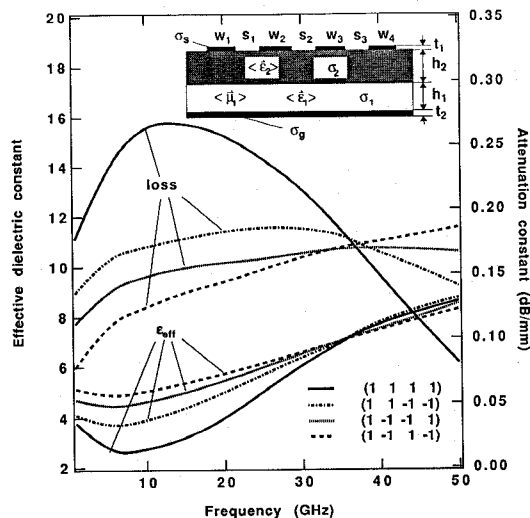
than the lower one (as in MMIC applications where the upper layer acts as a buffer), then the effective permittivities do no longer follow the pattern known from MIC's as is demonstrated in Fig. 3b. While the even mode still shows higher losses, even if different anisotropic ratios are considered, this mode now is lower in effective dielectric constant. When the two cases of Fig. 3a and Fig. 3b (curve 2) are compared, both effective permittivities are increased by the exchange of layers, but with considerably more emphasis on the odd mode due to the high permittivity of the now upper layer. Note that in this investigation, the tensor elements of the two anisotropic dielectrics have been chosen to be clearly distinguishable. Combinations of the above effects are likely to occur if higher anisotropic ratios are introduced. Fig. 3c shows the characteristic impedance for the structure of Fig. 3b. It is demonstrated that in spite of the change in effective permittivity, the even-mode characteristic impedance is still higher than that of the odd mode as required for contra-directional coupler applications.

The effective permittivities of a three-line structure on double-layered anisotropic substrate are shown in Fig. 4. The symbols +1 and -1 illustrate the direction of the current  $J_z$  (c.f. (6)), 0 denotes ground potential. Since the upper substrate layer is of higher permittivity, the mode with the highest effective dielectric constant is associated with the lowest losses, and vice versa (c.f. Fig. 3b). Note that there is an equal effective dielectric constant point near 47 GHz for all different modes. Although fairly high in frequency, this effect can be used in high-speed interconnect applications where equal phase velocities are required.



**Fig. 4** Effective dielectric constant and attenuation constant of three conductors on double-layered anisotropic substrate;  $w_1=w_3=1.0\text{mm}$ ,  $w_2=0.8\text{mm}$ ,  $s_1=s_2=1.0\text{mm}$ ,  $h_1=h_2=1.0\text{mm}$ ,  $\epsilon_{x1}=2.4$ ,  $\epsilon_{y1}=2.6$ ,  $\epsilon_{z1}=2.8$ ,  $\sigma_1=0.2\text{S/m}$ ,  $\epsilon_{x2}=\epsilon_{z2}=9.4$ ,  $\epsilon_{y2}=11.6$ ,  $\sigma_2=0.3\text{S/m}$ ,  $t_1=0.3\text{mm}$ ,  $t_2=0.5\text{mm}$ ,  $\sigma_s=\sigma_g=40\text{S}/\mu\text{m}$ .

At the example of a four-conductor configuration on double-layered bi-anisotropic substrate, Fig. 5 demonstrates that the point of equal phase velocities can be, first, reduced in frequency and, second, made to coincide with the point of almost identical losses. This effect of equal complex propagation constants of all fundamental modes involved seems to be extremely attractive for future high-speed interconnects and high-directivity contra-directional coupler applications.



**Fig. 5** Effective dielectric constant and attenuation constant of four conductors on double-layered bi-anisotropic substrate;  $w_1=w_4=1.0\text{mm}$ ,  $w_2=w_3=0.9\text{mm}$ ,  $s_1=s_3=1.0\text{mm}$ ,  $s_2=0.9\text{mm}$ ,  $h_1=1.0\text{mm}$ ,  $h_2=0.8\text{mm}$ ,  $\epsilon_{x1}=2.6$ ,  $\epsilon_{y1}=1.6$ ,  $\epsilon_{z1}=2.5$ ,  $\mu_{x1}=2.8$ ,  $\mu_{y1}=5.6$ ,  $\mu_{z1}=1$ ,  $\sigma_1=0.3\text{S/m}$ ,  $\epsilon_{x2}=\epsilon_{z2}=9.4$ ,  $\epsilon_{y2}=11.6$ ,  $\sigma_2=0.2\text{S/m}$ ,  $t_1=0.3\text{mm}$ ,  $t_2=0.5\text{mm}$ ,  $\sigma_s=\sigma_g=40\text{S}/\mu\text{m}$ .

#### IV. CONCLUSION

An extended spectral-domain immittance approach for the analysis of multiconductor systems on layered bi-anisotropic substrates is presented. The method takes into account all losses of conventional or high-Tc superconductors, of multilayered bi-anisotropic substrates and of imperfect ground metallizations. The algorithm is CPU-time efficient and, therefore, offers an attractive solution in CAD procedures for modern MMIC applications. It is demonstrated that by properly selecting substrate materials, a frequency of almost identical complex propagation constants for all fundamental modes in a multiconductor system can be obtained. This effect has a high potential for future applications in high-speed interconnects and high-directivity contra-directional couplers.

#### REFERENCES

- [1] T.C. Mu and T. Itoh, "Characteristics of multiconductor, asymmetric, slow-wave microstrip transmission lines", *IEEE Trans. Microwave Theory Tech.*, vol. 34, pp. 1471-1477, Dec. 1986.
- [2] J.P.K. Gilb and C.A. Balanis, "Asymmetric, multi-conductor low-coupling structures for high-speed, high-density digital interconnects", *IEEE Trans. Microwave Theory Tech.*, vol. 39, pp. 2100-2106, Dec. 1991.
- [3] N.G. Alexopoulos, "Integrated-circuit structures on anisotropic substrate", *IEEE Trans. Microwave Theory Tech.*, vol. 33, pp. 847-881, Oct. 1985.
- [4] V.K. Tripathi and H. Lee, "Spectral-domain computation of characteristic impedances and multiport parameters of multiple coupled microstrip lines", *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 215-221, Jan. 1989.
- [5] M.R.D. Maia, A.G. D'Assunção and A.J. Giarola, "Dynamic analysis of microstrip lines and finlines on uniaxial anisotropic substrates", *IEEE Trans. Microwave Theory Tech.*, vol. 35, pp. 881-886, Oct. 1987.
- [6] M. Horno, F.L. Mesa, F. Medina and R. Marques, "Quasi-TEM analysis of multilayered, multiconductor coplanar structures with dielectric and magnetic anisotropy including substrate losses", *IEEE Trans. Microwave Theory Tech.*, vol. 38, pp. 1059-1068, Aug. 1990.
- [7] A.A. Mostafa, C.M. Krowne and K.A. Zaki, "Numerical spectral matrix method for propagation in anisotropic layered media", in *1987 IEEE MTT-S Intl. Microwave Symp. Dig.*, pp.311-314.
- [8] C.M. Krowne, "Relationship for Green's function spectral dyadics involving anisotropic imperfect conductors imbedded in layered anisotropic media", *IEEE Trans. Antennas Propagat.*, vol. 37, pp. 1207-1211, Sep. 1989.
- [9] J.M. Pond, C.M. Krowne and W.L. Carter, "On the application of complex resistive boundary conditions to model transmission lines consisting of very thin superconductors", *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 181-190, Jan. 1989.
- [10] Z. Cai and J. Bornemann, "Generalized spectral-domain analysis for multilayered complex media and high-Tc superconductor applications", *IEEE Trans. Microwave Theory Tech.*, vol. 40, Dec. 1992.