ELEC 310. Lecture 4
Basic concepts in DT signals and DT systems

Textbook: sections 2.1, 2.2

Today

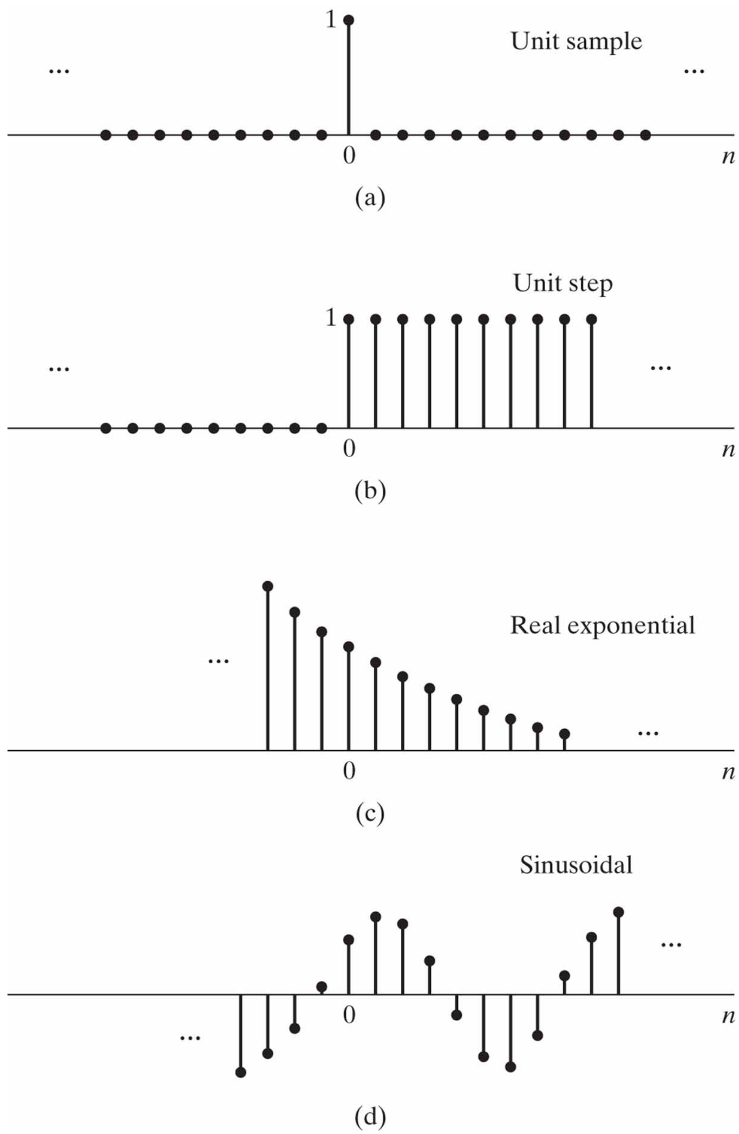
- Elementary DT signals
  - The DT unit impulse and the unit step functions
  - Exponential DT signals

- Discrete-time systems
  - Properties and classes of systems

Elementary signals

- Are basic building blocks for synthesizing/ decomposing a wide range of more complex signals
- What types of signals should be chosen as primitives?
  - It depends on the class of signals/systems one wants to analyze/design/simulate
  - ELEC 310 deals with Linear Time Invariant Systems
  - For this class of systems **exponential and sinusoidal functions** are shape-invariant
  - It is useful to consider these functions as **elementary building blocks**

Figure 2.3 Some basic sequences. The sequences shown play important roles in the analysis and representation of discrete-time signals and systems.



Non-periodic elementary signals

- Building blocks for constructing and representing signals and systems
- The unit impulse (unit sample):

$$\delta[n] = \begin{cases} 0 & \text{if } n \neq 0 \\ 1 & \text{if } n = 0 \end{cases}$$

- The unit step:

$$u[n] = \begin{cases} 0 & \text{if } n < 0 \\ 1 & \text{if } n \geq 0 \end{cases}$$

Relationships between DT unit impulse and DT unit step

- The DT unit impulse is the first difference of the DT unit-step

$$\delta[n] = u[n] - u[n-1]$$

- Conversely, the DT unit step is the running sum of the unit sample

$$u[n] = \sum_{m=-\infty}^{n} \delta[m]$$

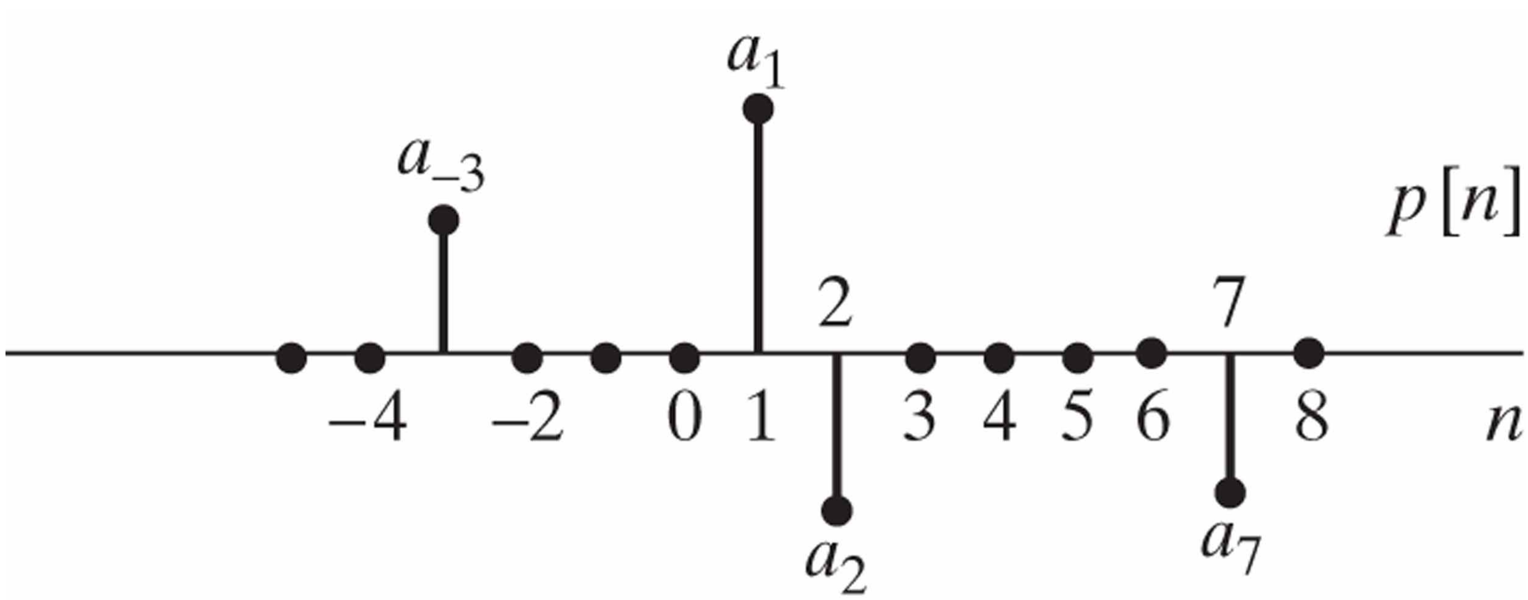
$$u[n] = \sum_{k=0}^{\infty} \delta[n-k]$$

Sampling property of the unit impulse sequence

$$x[n]\delta[n] = x[0]\delta[n]$$

$$x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0];$$

Representation of DT signals as sums of scaled and shifted impulses



CT unit impulse and unit step functions

- CT unit step function: $$u(t) = \begin{cases} 0 & \text{if } t < 0, \\ 1 & \text{if } t \geq 0. \end{cases}$$

- CT unit step function is the running integral of the unit impulse:

$$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau$$

- Conversely, the unit impulse can be thought as the first derivative of the unit step

$$\delta(t) = \frac{du(t)}{dt}$$

CT unit step and unit impulse functions (cont'd)

- u(t) is discontinous at t=0 and formally not differentiable;
- We can imagine a limit process

$$\delta_\Delta(t) = \frac{du_\Delta(t)}{dt} \xrightarrow[\Delta \to 0]{} \delta(t)$$

- with DT signals we do not need continuity, derivability etc.

Exponential signals

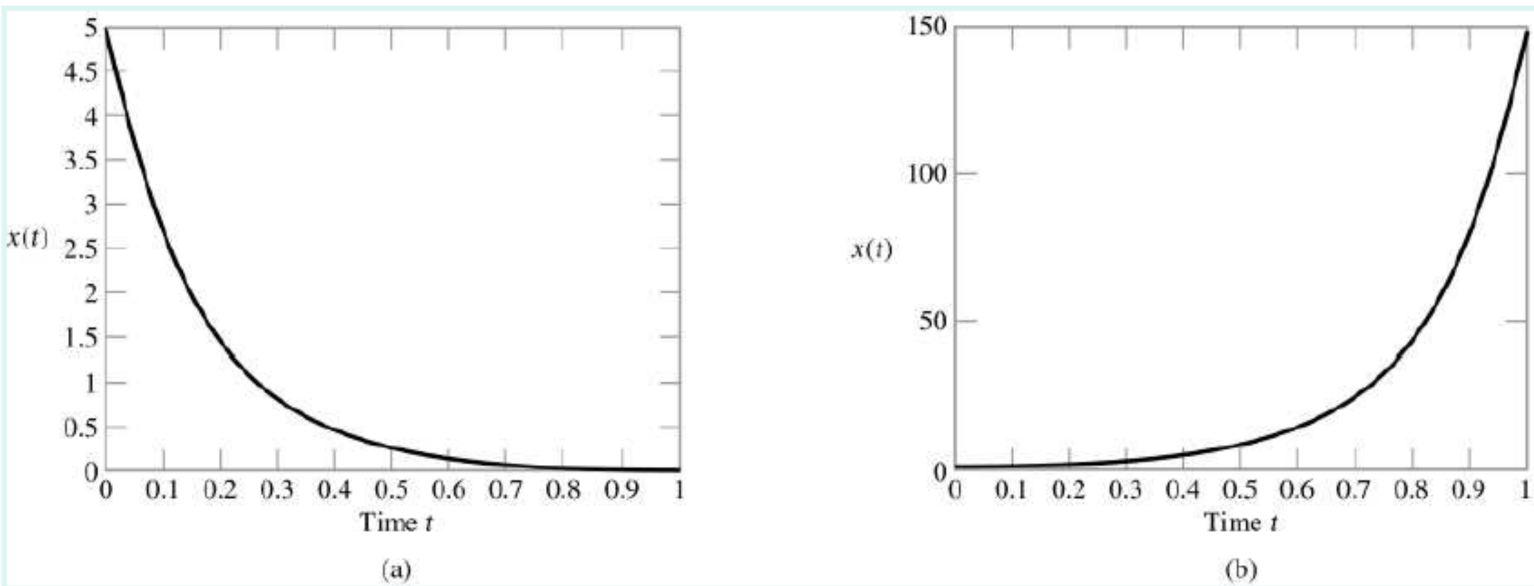
- Complex exponential signal:
- $x[n]=A\alpha^n$, A and α complex
- Some cases of particular interest:
  - A real, α real: real exponentials
  - $|\alpha|=1$; $x[n]$ is periodic and sinusoidal
  - A complex, α complex: general complex exponential signals

CT real exponential signals

$$x(t) = Ce^{at}, C, a \in R$$

$a<0$ decaying exponential
(step response of an RC circuit)

$a>0$ growing exponential
(chain reaction)

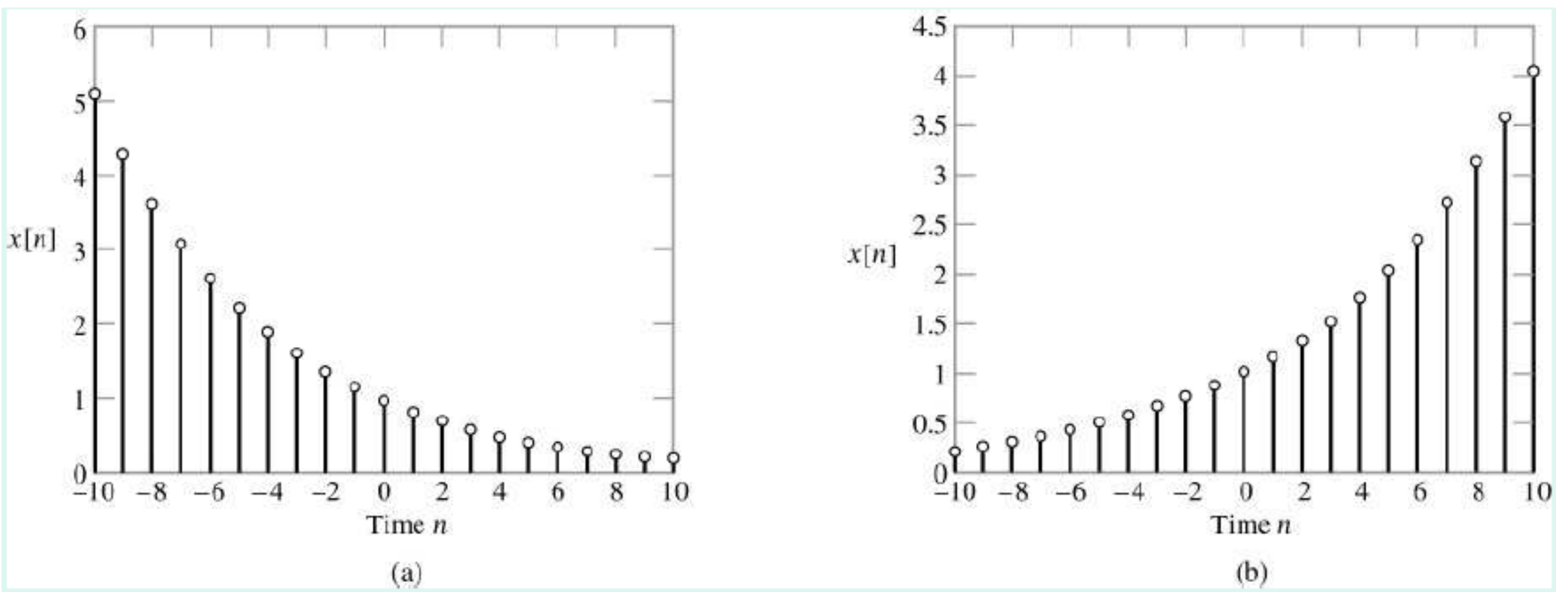


DT real exponential signals

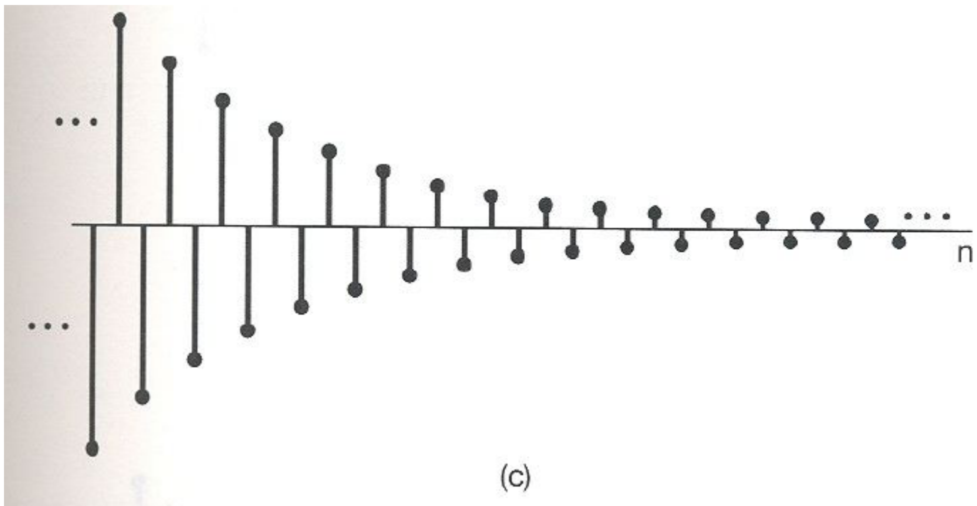
$$x[n] = Ce^{an} = C\alpha^{n}$$

0<α<1
Decaying exponential

α>1
Growing exponential

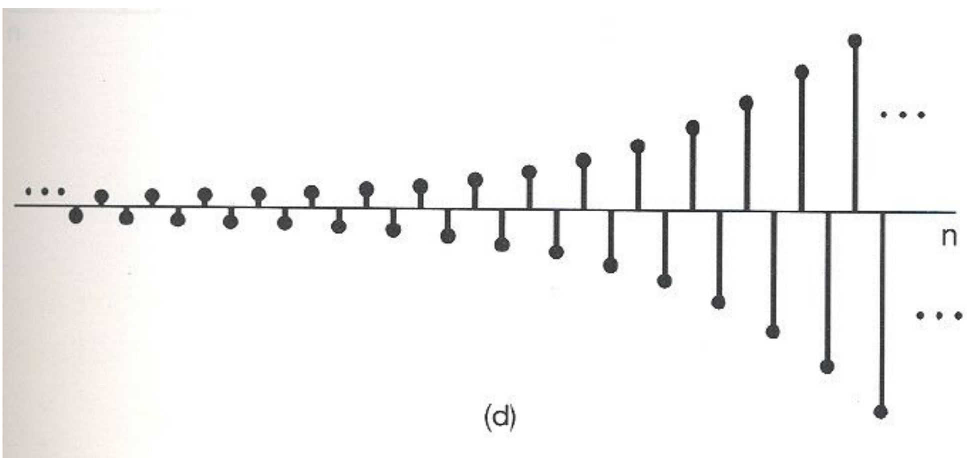


DT real exponential signals (cont'd)



$-1<\alpha<0$

$$x[n] = Ce^{an} = C\alpha^{n}$$



$\alpha<-1$

DT periodic signals: exponential and sinusoidal forms

$x[n]=A\alpha^n$ with $|\alpha|=1$

We can express x[n] as a sum of sinusoidal signals…

The exponential and sinusoidal forms of periodic signals are related and it is very useful to know how to translate one form into the other.

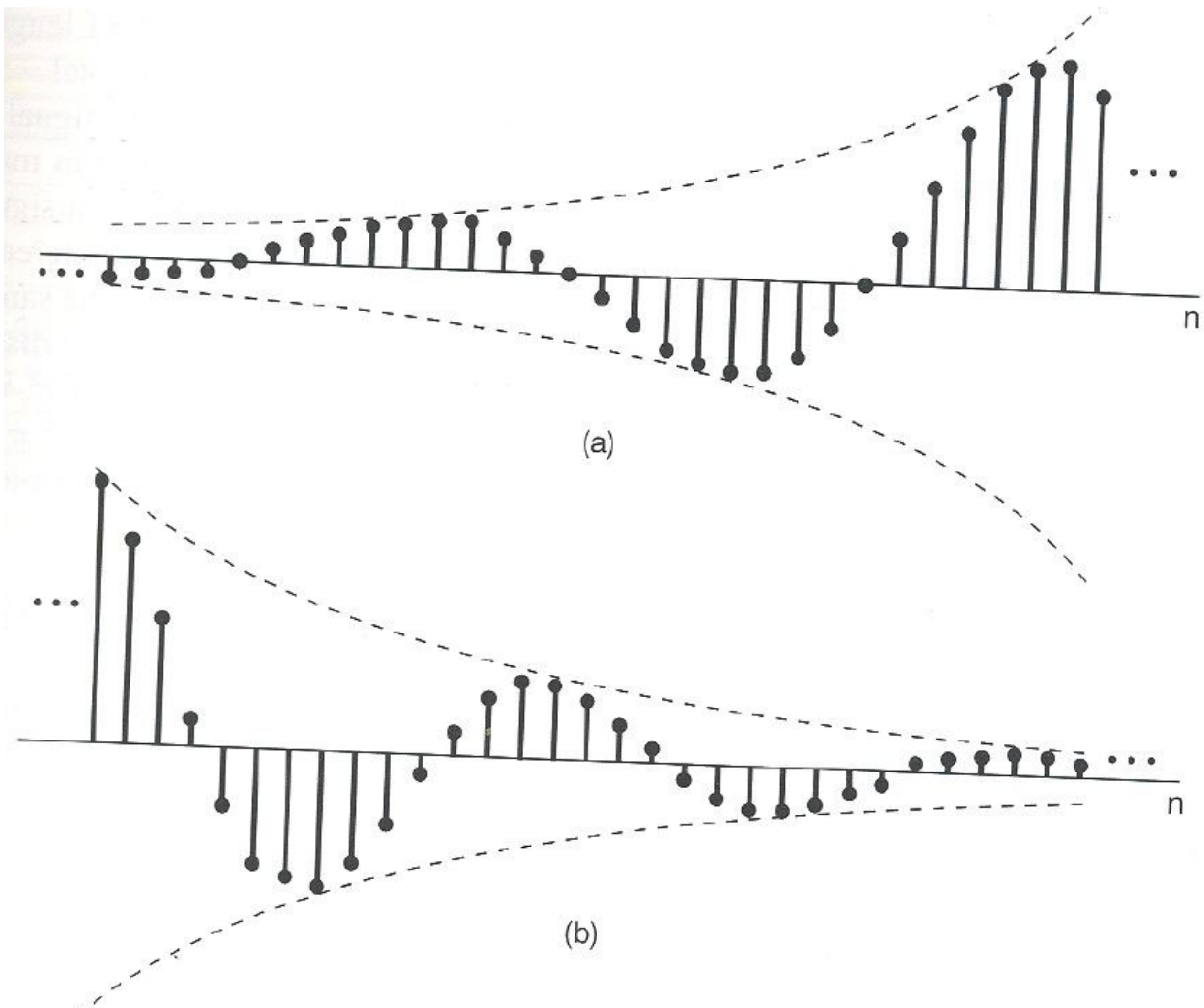
Given $x[n] = A\cos(\omega_0 n + \phi)$
we want to express it as a sum of complex exponentials…

DT sinusoidal signals (cont'd)

- Periodic exponentials will serve for building representations of other signals
- We will use sets of harmonics, all of which are periodic with a common period T_0

$$\varphi_k[n] = e^{jk\omega_0 N}, \quad k = \pm 1, \pm 2, \pm 3 \text{ etc}$$

General complex exponential signals



Discrete-time systems

- We represent a discrete-time system as a transformation that maps an input sequence $x[n]$ into a unique output sequence $y[n]$.

- Study examples 2.2, 2.3 from textbook

Classes of systems

- Classes of systems are defined by placing constraints on the properties of the transform T.

- Memoryless systems
- Linear systems (examples 2.5, 2.6)
- Time-invariant systems (example 2.7, 2.8)
- Causal systems (example 2.9)
- Stable systems (example 2.10)

Testing system properties

2.23 a. Determine if the system given by

$$T(x[n]) = (\cos \pi n) x[n]$$

is:

a) stable; b) causal; c) linear; d) time-invariant