ECE 405/511 Assignment 1 2026 Solutions

1. $p=0.1$ probability of a bit in error

$1-p=0.9$ probability of a bit receved correctly

$(1-p)^8 = .9^8 = .4305$ probability of 8 bits received correctly

$1-(1-p)^8 = 1-.4305 = .5695$ probability of an undetected error

with an even parity check the probability of an udetected error for an 8 bit codeword is

$$\binom{8}{2} p^2 (1-p)^6 + \binom{8}{4} p^4 (1-p)^4 + \binom{8}{6} p^6 (1-p)^2 + \binom{8}{8} p^8$$

$$= 28 \times 5.314 \times 10^{-3} + 70 \times 6.561 \times 10^{-5} + 28 \times 8.1 \times 10^{-7} + 10^{-8}$$

$$= .1534$$

2. binary code codewords of length $n = 23$

$t = 3$

$$P(E) = \sum_{i=4}^{23} \binom{23}{i} p^i (1-p)^{23-i}$$

$$P(C) = 1 - P(E) = \sum_{i=0}^{3} \binom{23}{i} p^i (1-p)^{23-i}$$

this is the probability of correct decoding (3 or fewer errors)

$$P(C) = (1-p)^{23} + 23p(1-p)^{22} + 253p^2(1-p)^{21} + 1771p^3(1-p)^{20}$$

for the BER, assuming $p \ll \frac{1}{2}$

the most likely error event is $t+1 = 4$ errors

for this event, the worst case is that decoding introduces 3 additional errors

so $BER = \frac{4+3}{23} P(E)$

p	P(E)	BER
.1	.1927	.05866
.01	7.605×10^{-5}	2.315×10^{-5}
.001	8.721×10^{-9}	2.654×10^{-9}
.0001	8.842×10^{-13}	2.691×10^{-13}

3. Dimension of the binary vector space spanned by

001010
101000
001100
100010
011111

by inspection, vectors 1, 2, and 4 sum to 000000

- thus these vectors are not linearly independent
- removing one of these rows gives 4 vectors that are linearly independent

$\therefore$ the dimension of the vector space is 4

4. The vector space S spanned by

$$\begin{bmatrix} 11100 \\ 01110 \\ 00111 \end{bmatrix}$$

has dimension 3

$\dim(S) + \dim(S^{\perp}) = 5 \rightarrow \dim(S^{\perp}) = 2$

the dual space has dimension 2 so two linearly independent vectors orthogonal to the vectors in S are required to construct a basis for $S^{\perp}$

by inspection 11011 and 01101 are orthogonal to the vectors in S

thus a basis for $S^{\perp}$ is $\begin{bmatrix} 11011 \\ 01101 \end{bmatrix}$

$S^{\perp} =$ 00000
11011
01101
10110

5. $C = \{(00100), (10010), (01001), (11111)\}$

a) minimum distance – find the Hamming distance between all pairs of codewords

00100	00100	00100	10010	10010	01001
10010	01001	11111	01001	11111	11111
10110	01101	11011	11011	01101	10110
3	3	4	4	3	3

the smallest difference is 3 so $d_{min} = 3$

b) the code can detect $v = d_{min} - 1 = 2$ errors

c) the code can correct $t = \left\lfloor \frac{d_{min}-1}{2} \right\rfloor = 1$ error

d) the code is NOT linear

$$\begin{array}{r} 00100 \\ +\ 10010 \\ \hline 10110 \end{array}$$

is not a codeword so the closure property is violated

6. Find the maximum likelihood codewords

a) r=10110

codeword	distance from r
00100	2
10010	1
01001	5
11111	2

∴ 10010 is the maximum likelihood codeword

b) r=01010

codeword	distance from r
00100	3
10010	2
01001	2
11111	3

∴ 10010 or 01001 can be chosen as the maximum likelihood codeword

7. The parity check matrix is

$$H = \begin{array}{cc} \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} & 4 \\ 7 & \end{array}$$

from this matrix $n=7$ and $n-k=4$

$\therefore$ the length is $n=7$ and the dimension is $k=3$

the minimum distance is the minimum number of columns of H that sum to 0

- no two columns are the same
  $\therefore d_{min} > 2$

- all columns have weight 1 or 3 so three columns cannot sum to 0
  $\therefore d_{min} > 3$

- columns 1, 2, 3 and 5 sum to 0

$\therefore d_{min} = 4$

generator matrix

permute the columns of H

$$\begin{array}{cc}
 & \begin{array}{ccccccc} 3 & 5 & 6 & 2 & 1 & 4 & 7 \end{array} \\
H' = & \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} = \left[I_4 \; P^T \right]
\end{array}$$

H' is in systematic form so

$$G' = \left[P \; I_3 \right] = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

reverse the permutation to get G

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

the codewords are all combinations of the rows of G

codeword	weight
0000000	0
1110100	4
0101110	4
1011010	4
0110011	4
1000111	4
0011101	4
1101001	4

All nonzero codewords have weight 4

$\therefore d_{min} = 4$