ECE 405/511 Assignment 2 2026 Solutions

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

(a) systematic form is $[I \; P]$ or $[P \; I]$

transform G by row and column operations

add rows 1 and 3

$$G' = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

permute columns 2 and 5
and 3 and 6

$$G'' = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

this gives the form $[I \; P]$

to get $[P \; I]$ permute the columns

of the matrix

(b) for G in the form $[I \; P]$

the parity check matrix is

$$H = [P^T \; I]$$

$$= \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

for G in the form $[P \; I]$

the parity check matrix is

$$H = [I \; P^T]$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

2. the binary Hamming codes have parameters

$n = 2^m - 1 \quad k = 2^m - 1 - m \quad n-k = m \quad d_{min} = 3$

the extended Hamming codes have parameters

$n = 2^m \quad k = 2^m - 1 - m \quad n-k = m+1 \quad d_{min} = 4$

3. dimension $n-k = m = 4$

(8,4,4) extended Hamming code

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

there are $2^4 = 16$ codewords

00000000	01100110	10110010
10000111	11100001	11010100
01001101	00011110	01111000
11001010	10011001	11111111
00101011	01010011	
10101100	00110101	

all codewords have even weight

there are 14 codewords of weight 4
and 1 codeword of weight 8

$\therefore$ $d_{min} = 4$ as in Problem 2

for a code to be self-dual requires that
the rows of the generator matrix are
orthogonal

taking the inner products

$(1000\ 0111) \circ (0100\ 1101) = 0$
$(1000\ 0111) \circ (0010\ 1011) = 0$
$(1000\ 0111) \circ (0001\ 1110) = 0$

$(0100\ 1101) \circ (0010\ 1011) = 0$
$(0100\ 1101) \circ (0001\ 1110) = 0$

$(0010\ 1011) \circ (0001\ 1110) = 0$

$\therefore$ the code is self-dual

4. $$H = \begin{array}{cc} \begin{bmatrix} 1&0&0&0&1&1\\ 0&1&0&1&0&1\\ 0&0&1&1&1&1 \end{bmatrix} & 3 \\ 6 & \end{array}$$

from this matrix $n=6$ and $n-k=3$

so $k=3$

H has the form $[I\ P^T]$ so

so $G=[P\ I] = \begin{bmatrix} 0&1&1&1&0&0\\ 1&0&1&0&1&0\\ 1&1&1&0&0&1 \end{bmatrix}$

code word	weight
000000	0
011100	3
101010	3
111001	4
110110	4
010011	3
100101	3
001111	4

the minimum nonzero weight is 3 so $d_{min}=3$

since $d_{min} = 3$ $t = 1$

$\therefore$ the correctable error patterns are

$e_0 = 000000$
$e_1 = 100000$
$e_2 = 010000$
$e_3 = 001000$
$e_4 = 000100$
$e_5 = 000010$
$e_6 = 000001$

the corresponding syndromes are

$e_0 H^T = 000 = S_0$

$e_1 H^T = 100 = S_1$

$e_2 H^T = 010 = S_2$

$e_3 H^T = 001 = S_3$

$e_4 H^T = 011 = S_4$

$e_5 H^T = 101 = S_5$

$e_6 H^T = 111 = S_6$

there is one syndrome remaining

$S_7 = 110$

this can be associated with

$e_7 = 110000$

for $r = 110111$ $\quad s = rH^T = 111 = S_6$

so $c = r + e_6 = $

$$\begin{array}{r} 110111 \\ +\ 000001 \\ \hline 110110 \end{array}$$

for $r = 011001$ $\quad s = rH^T = 100 = S_1$

so $c = r + e_1 = $

$$\begin{array}{r} 011001 \\ +\ 100000 \\ \hline 111001 \end{array}$$

5. Determine using the Hamming bound if a (9,4,5) binary code can exist

$$K \le n - \left\lceil \log_2 \left(\sum_{i=0}^{t} \binom{n}{i} \right) \right\rceil$$

$n = 9 \quad t = 2$

$$\sum_{i=0}^{2} \binom{9}{i} = \binom{9}{0} + \binom{9}{1} + \binom{9}{2} = 46$$

$\log_2 46 = 5.52$

$K \le 9 - \lceil 5.52 \rceil = 9 - 6 = 3$

$\therefore$ a (9,4,5) code cannot exist

6. Gilbert-Varshamov bound

$$k \geq n - \left\lfloor \log_2 \left(\sum_{j=0}^{d-2} \binom{n-1}{j} \right) \right\rfloor - 1$$

(9,2,6) binary code

$n = 9 \qquad d = 6$

$$\sum_{j=0}^{4} \binom{8}{j} = \binom{8}{0} + \binom{8}{1} + \binom{8}{2} + \binom{8}{3} + \binom{8}{4}$$

$$= 163$$

$\log_2 163 = 7.35$

$k \geq 9 - \lfloor 7.35 \rfloor - 1 = 9 - 8 = 1$

$\therefore$ the bound does not verify that the code exists

However such a code can be constructed

$$G = \begin{bmatrix} 111 & 111 & 000 \\ 000 & 111 & 111 \end{bmatrix}$$

the code words are

000 000 000
111 111 000
000 111 111
111 000 111

7. the parameters of the Hamming codes are

$$\left(\frac{q^m - 1}{q-1}, \frac{q^m-1}{q-1} - m, 3 \right)$$

$$q^{n-k} = q^m$$

the sphere volume is

$$Vol(n,1) = \sum_{i=0}^{1} \binom{n}{i} (q-1)^i$$

$$= n(q-1) + 1 = \frac{q^m-1}{q-1}(q-1) + 1 = q^m$$

$\therefore$ the sphere volume is equal to q^{n-k}
so the codes are perfect