ECE 405/511 Assignment 4 Spring 2026 SOLUTIONS

1. $g(x) = x^5 + x^4 + x^2 + 1$

(a) $m(x) = x^2$

$$c(x) = m(x)\,g(x)$$

$$= x^2(x^5 + x^4 + x^2 + 1)$$

$$= x^7 + x^6 + x^4 + x^2$$

$c = 0010101100000000$

(b) $m(x) = x^8 + x^7 + x^6 + x^5 + x^4$

$$c(x) = m(x)\,g(x)$$

$$= (x^8 + x^7 + x^6 + x^5 + x^4)(x^5 + x^4 + x^2 + 1)$$

$$\begin{aligned} = x^{13} + x^{12} + x^{11} + x^{10} + x^9 \\ + x^{12} + x^{11} + x^{10} + x^9 + x^8 \\ + x^{10} + x^9 + x^8 + x^7 + x^6 \\ + x^8 + x^7 + x^6 + x^5 + x^4 \end{aligned}$$

$$= x^{13} + x^{10} + x^9 + x^8 + x^5 + x^4$$

$c = 0000111011110010$

2. $g(x) = x^5 + x^4 + x^2 + 1$

$n = 15 \quad k = 10 \quad n - k = 5$

(a) $m(x) = x^2$

$$m(x)\, x^{n-k} = x^7$$

$$\begin{array}{r|l}
 & x^2 + x + 1 \\
\hline
x^5 + x^4 + x^2 + 1 & x^7 \\
 & \underline{x^7 + x^6 + x^4 + x^2} \\
 & \quad x^6 + x^4 + x^2 \\
 & \quad \underline{x^6 + x^5 + x^3 + x} \\
 & \qquad x^5 + x^4 + x^3 + x^2 + x \\
 & \qquad \underline{x^5 + x^4 \qquad + x^2 + 1} \\
 & \qquad\qquad x^3 + x + 1
\end{array}$$

$d(x)$

$$c(x) = m(x)x^{n-k} - d(x)$$

$$= x^7 + x^3 + x + 1$$

$c = 110100010000000$

(b) $m(x) = x^8 + x^7 + x^6 + x^5 + x^4$

$m(x)\,x^{n-k} = x^{13} + x^{12} + x^{11} + x^{10} + x^9$

$$
\begin{array}{r}
x^8 + x^6 + x^5 + x^2 + 1 \\
x^5 + x^4 + x^2 + 1 \;\big)\; x^{13} + x^{12} + x^{11} + x^{10} + x^9 \\
\underline{x^{13} + x^{12} + x^{10} + x^8} \\
x^{11} + x^9 + x^8 \\
\underline{x^{11} + x^{10} + x^8 + x^6} \\
x^{10} + x^9 + x^6 \\
\underline{x^{10} + x^9 + x^7 + x^5} \\
x^7 + x^6 + x^5 \\
\underline{x^7 + x^6 + x^4 + x^2} \\
x^5 + x^4 + x^2 \\
\underline{x^5 + x^4 + x^2 + 1} \\
1
\end{array}
$$

$d(x)$

$c(x) = m(x)\,x^{n-k} + d(x)$

$= x^{13} + x^{12} + x^{11} + x^{10} + x^9 + 1$

$c = 100000000111110$

3. $g(x) = x^5 + x^4 + x^2 + 1$

a) $r(x) = x^{10}$ $\quad$ $s(x) = r(x)/g(x)$

$$
\begin{array}{r}
x^5 + x^4 + x^3 + x + 1 \\
x^5 + x^4 + x^2 + 1 \;\big)\; x^{10} \qquad\qquad\qquad\qquad \\
\underline{x^{10} + x^9 + x^7 + x^5} \qquad\qquad\quad \\
x^9 + x^7 + x^5 \qquad\qquad\quad \\
\underline{x^9 + x^8 + x^6 + x^4} \qquad\qquad \\
x^8 + x^7 + x^6 + x^5 + x^4 \qquad \\
\underline{x^8 + x^7 \quad + x^5 \quad + x^3} \qquad \\
x^6 + x^4 + x^3 \qquad \\
\underline{x^6 + x^5 + x^3 + x} \\
x^5 + x^4 + x \\
\underline{x^5 + x^4 + x^2 + 1} \\
x^2 + x + 1
\end{array}
$$

$s(x) = x^2 + x + 1$

$s = 11100$

b) $r(x) = x^8 + x^6 + x + 1$ $\qquad$ $s(x) = r(x)/g(x)$

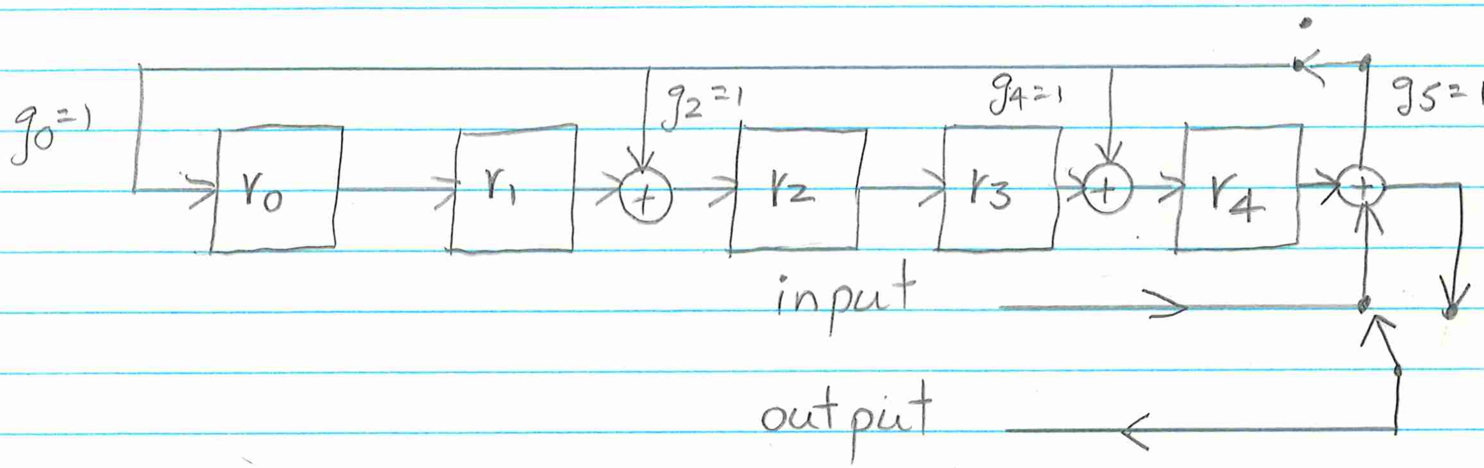
$$
\begin{array}{r l}
 & x^3 + x^2 + 1 \\
x^5 + x^4 + x^2 + 1 \;\big)\!\!\! & x^8 + x^6 + x + 1 \\
 & \underline{x^8 + x^7 + x^5 + x^3} \\
 & x^7 + x^6 + x^5 + x^3 + x + 1 \\
 & \underline{x^7 + x^6 + x^4 + x^2} \\
 & x^5 + x^4 + x^3 + x^2 + x + 1 \\
 & \underline{x^5 + x^4 + x^2 + 1} \\
 & x^3 + x
\end{array}
$$

$s(x) = x^3 + x$

$s = 01010$

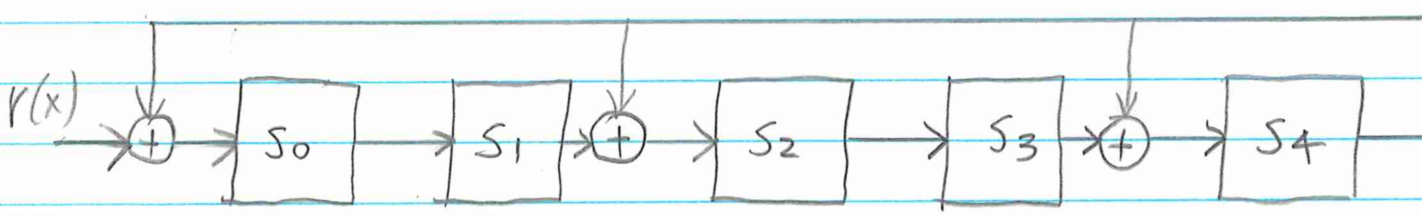
4.

systematic encoder $g(x) = x^5 + x^4 + x^2 + 1$




syndrome computation

divide $r(x)$ by $g(x)$



5 crc-6 detect all odd-weight error patterns

$g(x) = (x+1)\, b(x)$

the best choice of $b(x)$ is a primitive polynomial of degree 5

choose $b(x) = x^5 + x^2 + 1$

$g(x) = (x+1)(x^5 + x^2 + 1)$

$= x^6 + x^5 + x^3 + x^2 + x + 1$

$b(x)$ generates a $(31, 26, 3)$ Hamming code

$\therefore$ $g(x)$ generates a $(31, 25, 4)$ cyclic code

burst error detection $\deg(g(x)) = n-k = 6 = r$

(a) 1.0 (b) 1.0 (c) $1 - 2^{1-r} = 1 - 2^{-5} = 31/32$

(d) $1 - 2^{-r} = 1 - 2^{-6} = 63/64$ (e) $63/64$

6. binary narrow-sense BCH code

length $n=15$ design distance 3

$\Rightarrow$ 2 consecutive powers of an element of order 15 in $GF(16)$

let $\alpha$ be a root of $p(x)=x^4+x+1$
a primitive polynomial of degree 4 over $GF(2)$

$\rightarrow$ narrow-sense so the consecutive powers are $\alpha, \alpha^2$

from the cyclotomic coset $\{1,2,4,8\}$
both powers of $\alpha$ are roots of $p(x)$

a) $g(x)=x^4+x+1$

b) rate is $\frac{11}{15}=.733$

c) $h(x)=\dfrac{x^n+1}{g(x)} = x^{11}+x^8+x^7+x^5+x^3+x^2+x+1$

$h^*(x)= x^{11}+x^{10}+x^9+x^8+x^6+x^4+x^3+1$

$$G = \begin{bmatrix} 11001 & & & & \\ & 11001 & & & 0 \\ & & 11001 & & \\ & & & \ddots & \\ & 0 & & & \\ & & & & 11001 \end{bmatrix} \text{ 11 rows}$$

15 columns

$$H = \begin{bmatrix} 1&0&0&1&1&0&1&0&1&1&1&1&0&0&0 \\ 0&1&0&0&1&1&0&1&0&1&1&1&1&0&0 \\ 0&0&1&0&0&1&1&0&1&0&1&1&1&1&0 \\ 0&0&0&1&0&0&1&1&0&1&0&1&1&1&1 \end{bmatrix}$$

7. binary BCH code of length $n=15$
design distance 4

a) narrow-sense $\Rightarrow$ 3 consecutive powers of $\alpha$ beginning with $\alpha^1$

the cyclotomic cosets are

$\{0\}, \{1,2,4,8\}, \{3,6,12,9\}, \{5,10\}, \{7,14,13,11\}$

roots of $g(x)$ are $\alpha^1, \alpha^2, \alpha^3$

$\therefore\ g(x) = M_1(x)\, M_3(x)$

$= (x^4+x+1)(x^4+x^3+x^2+x+1)$

$= x^8+x^7+x^6+x^4+1$

b) from the cyclotomic cosets, choose three consecutive numbers that give $g(x)$ with the lowest degree

$\alpha^0$ has a degree 1 minimal polynomial so choose

$\alpha^0, \alpha^1, \alpha^2$

$$g(x) = M_0(x)\, M_1(x)$$

$$= (x+1)(x^4+x+1)$$

$$= x^5 + x^4 + x^2 + 1$$

c) the code rate in (a) is $\frac{7}{15} = .467$

the code rate in (b) is $\frac{10}{15} = .667$

$\Rightarrow$ the code in (b) is more efficient

same design distance but a higher rate

$.667 > .467$