University of Victoria

Department of Electrical and Computer Engineering

ECE 511 **Error Control Coding** **Spring 2025**

Project Questions

Instructions

1. State any assumptions that may be necessary.
2. Show all details of your solution.
3. There are 10 problems of equal weight.

Provide solutions for the following problems

1. Given $\mathbf{u} = u_1, \ldots, u_m$ and $\mathbf{v} = v_1, \ldots, v_n$, define $(\mathbf{u}|\mathbf{v})$ as the vector $u_1, \ldots, u_m, v_1, \ldots, v_n$ of length $m+n$. Suppose that C_1 is a binary (n_1, M_1, d_1) code (with M_1 codewords) and that C_2 is a binary (n_2, M_2, d_2) code. Form a new code C_3 consisting of all vectors of the form $(\mathbf{u}|\mathbf{u}+\mathbf{v})$ where $\mathbf{u} \in C_1$ and $\mathbf{v} \in C_2$. Show that C_3 is a $(2n, M_1M_2, d)$ code with $d = \min\{2d_1, d_2\}$.

2. Let G_1 and G_2 be generator matrices of linear (n_1, k, d_1) and (n_2, k, d_2) codes, respectively. Show that the codes with generator matrices,

$$\begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix} \text{ and } (G_1 \ \ G_2)$$

are $(n_1+n_2, 2k, \min\{d_1, d_2\})$ and (n_1+n_2, k, d) codes, respectively, where $d \geq d_1 + d_2$.

3. Consider an $[n, k]$ binary linear code C whose generator matrix G contains no zero column. If all the codewords of C are arranged as rows of a $2^k \times n$ array,

   a) show that no column of the array contains only zeros.
   b) show that each column of the array contains 2^{k-1} zeros and 2^{k-1} ones.
   c) show that the set of all codewords with zeros in a particular location forms a subspace of C. What is the dimension of this subspace?
   d) show that the set of all codewords with even weight forms a subgroup of Z_2^n.

4. Consider the correspondence between letters and length 3 vectors over F_3 from associating a base 3 number to each letter via the following table, and then taking the digits in the base 3 number as entries in a vector.

Letter	Integer	Integer Base 3
blank	0	000
A	1	001
B	2	002
C	3	010
...	...	...
Z	26	222

Consider the parity check matrix H for a code C

$$H^T = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 0 & 1 \\ 2 & 2 & 2 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \\ 2 & 0 & 0 \end{bmatrix}.$$

a) Find a systematic matrix H for C. Then find a systematic generator matrix G for C.

b) Encode HELP using G.

c) Make a syndrome decoding table for C using only error vectors of weight 1. How many coset leaders would you need for complete maximum likelihood decoding?

d) Decode 100021, 220120, 020120, 012111, 000001, 2*2*01, 220121, 1**002. The * indicates an erasure.

5. Consider the $[2^m - 1, 2^m - m - 2]$ cyclic code C generated by $g(x) = (x+1)p(x)$ where $p(x)$ is a primitive polynomial of degree m.

   a) Show that C has minimum distance 4.

   b) An error pattern of the form

   $$e(x) = x^i + x^{i+1}$$

   is called a *double adjacent error pattern*. Show that no two double adjacent error patterns can have the same syndrome. Therefore the code is capable of correcting all single error patterns and all double adjacent error patterns.

6. Consider the double error correcting narrow-sense Reed-Solomon code of length 7 over GF(8) with generator polynomial $g(x) = (x-\alpha)(x-\alpha^2)(x-\alpha^3)(x-\alpha^4) = x^4 + \alpha^3 x^3 + x^2 + \alpha x + \alpha^3$. Use the Berlekamp-Massey algorithm to decode the received word $r = (\alpha^6 0 1 \alpha^3 1 0 0)$ where the lowest degree symbol is on the left. Draw the linear feedback shift register (LFSR) defined by the connection polynomial and verify that it generates the desired syndromes.

7. Determine a generator polynomial for a single-error-correcting 4-ary BCH code of length 63. The code should have the highest possible rate.

8. Construct a syndrome decoding table for the $(6,3,4)$ extended Reed-Solomon code over GF(4) given in the course slides. Decode the following received vectors

   (a) $r = (0\alpha 1 \alpha 1 \alpha^2)$

   (b) $r = (\alpha\alpha 1 \alpha^2 1 \alpha)$