

A Developmental Study of the Relationship Between Geometry and Kinematics in Drawing Movements

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Trajectory and kinematics of drawing movements are mutually constrained by functional relationships that reduce the degrees of freedom of the hand–arm system. Previous investigations of these relationships are extended here by considering their development in children between 5 and 12 years of age. Performances in a simple motor task—the continuous tracing of elliptical trajectories—demonstrate that both the phenomenon of *isochrony* (increase of the average movement velocity with the linear extent of the trajectory) and the so-called *two-thirds power law* (relation between tangential velocity and curvature) are qualitatively present already at the age of 5. The quantitative aspects of these regularities evolve with age, however, and steady-state adult performance is not attained even by the oldest children. The power-law formalism developed in previous reports is generalized to encompass these developmental aspects of the control of movement.

Two general frameworks are currently available to conceptualize the motor-control problem. Broadly, the two frameworks differ in the answer that they give to the question “Where do form and structure come from?” According to the motor-program view (e.g., Schmidt, 1988), form and structure in a movement come from a central abstract representation of the intended result. Thus, making a movement entails a detailed mapping of this representation into an appropriate motor plan. The competing task-dynamic view (e.g., Kugler, 1989; Saltzman & Kelso, 1987) emphasizes instead the global morphogenetic power of the nonlinearities of the motor system. These two views need not be mutually exclusive. In fact, different classes of movements may require different conceptual frameworks for their analysis. At any rate (at least in the case of drawing movements considered here), an internal representation of the intended trajectory and kinematics must be available before execution. Consequently, much effort is being devoted to understanding how this internal model is translated into a set of motor commands. In this respect, one basic conceptual difficulty is because the geometry of the arm–forearm–hand system affords a large number of degrees of freedom. Provided that there are no constraints in the workspace, the only trajectory that matters in many manual tasks is the more distal one, the so-called endpoint trajectory.

Geometrical considerations show that there are generally an infinite number of different sets of rotations at the intervening joints that result in identical endpoint trajectories. In principle, the nervous system could implement any of these valid solutions and even pick up a different solution every time a movement is repeated. In practice, the system does not seem to take advantage of such a freedom because only one specific set of rotations at the articular joints is consistently selected for generating a given trajectory (Soechting, Lacquaniti, & Terzuolo, 1986). Most attempts to explain how the motor system effectively controls this biomechanical system assume that the excess degrees of freedom are dissipated by some internal constraints (cf. Whiting, 1984); they differ only in the approach for identifying these constraints.

Constraining Principles

Top-down strategies seek to derive a set of constraints from some plausible a priori intuition about control optimality. In particular, attempts have been made to demonstrate that the solution actually adopted minimizes some global cost function. Hypotheses on the nature of this cost function include (a) the total energy required to implement the motor act (e.g., Nelson, 1983), (b) the global sense of effort (Hasan, 1986), (c) the average derivative of the acceleration (Flash & Hogan, 1985; Hogan, 1984; Wann, Nimmo-Smith, & Wing, 1988), and (d) the rate of change of the articular torques (Uno, Kawato, & Suzuki, 1989). Wann (1987) contrasted several cost-minimizing models on the basis of handwriting movements in children. Alternatively, the bottom-up approach attempts first to expose some constraints at the level of the measureable aspects of the movement and then speculates post-factum on the possible significance of these empirical findings from the point of view of optimality. Soechting et al. (1986; Soechting & Terzuolo, 1986, 1987) pursued this route by showing that when we trace elliptical or extemporaneous trajectories in space, the uniqueness of the solution results from an underlying correlation among the joint angles meas-

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ured in the Eulerian system of reference (yaw, pitch, and roll). Another possibility in the same vein was explored in a series of previous investigations (Lacquaniti, Terzuolo, & Viviani, 1983, 1984; Viviani & McCollum, 1983; Viviani & Terzuolo, 1980, 1982). Instead of considering joint angles, we concentrated on the endpoint motion itself and particularly on the relationships between the trajectory and kinematics of this motion.

Two such relationships are directly relevant to the experiments reported here. The first was hinted at almost a century ago by Binet and Courtier (1893) and Jack (1895), who noted that the velocity of upper-limb movements covaries with the curvature of the trajectory (see also Derwort, 1938). We provide the first quantitative description of this regularity through the so-called *two-thirds power law*, which states that at all points of a movement trajectory that are sufficiently removed from an inflection (in practice, when the curvature is greater than 0.1), the angular velocity is proportional to the $2/3$ power of the curvature, or equivalently that the tangential velocity is proportional to the cubic root of the radius of curvature. The second relationship of interest here exists between the size of the trajectory and the average velocity. Qualitative descriptions of a positive correlation between these two movement aspects can again be found in some early studies of human movements (Binet & Courtier, 1893; Derwort, 1938; Freeman, 1914). Moreover, in the special case of point-to-point rectilinear movements, such a correlation is implicit in Fitts's Law (Fitts, 1954; Michel, 1971). We extended these early observations to the general case of curvilinear trajectories and provided a quantitative formulation of the phenomenon (termed *isochrony*) that is valid for both periodic and aperiodic movements. The two-thirds power law and isochrony can be construed as rules for prescribing two complementary aspects of endpoint velocity, namely its instantaneous modulations and its average value. Because endpoint velocity is uniquely specified by the angular positions and velocities at the intervening joints, these empirical relationships represent a set of constraints among the corresponding torques that restrict the range of possible solutions to the problem of specifying the set of motor commands to be delivered to execute a given trajectory. Note that the two-thirds power law not only constrains the production of extemporaneous movements but also limits our capability to reproduce external models by pursuit tracking: Several studies have shown (Viviani, 1988; Viviani, Campadelli, & Mounoud, 1987; Viviani & Mounoud, 1990) that it is almost impossible to follow accurately dynamic target models that do not comply with this rule.

So far, no bottom-up strategy has been pursued far enough to afford a true solution to the degrees-of-freedom problem. Neither the correlation among joint angles nor the relationship between trajectory and kinematics can be construed as first principles of motor organization. Indeed, they are both likely to be the result of common underlying principles acting at a higher hierarchical level. We cover this point as well as the possible connections between bottom-up and top-down solutions in the Discussion section of this article. We now consider an issue that arises in conjunction with the translation of the internal model into a set of motor commands.

Computational Complexity

The trajectory and kinematics of hand movements are determined jointly by both the time course of the active torques at the articulations and by the masses that are set in motion. It is well known (cf. Brady, Hollerbach, Johnson, Lozano-Pérez, & Mason, 1985; Soechting, 1983) that as soon as we take into account inertial torques and couplings, the equations governing the movement of a system with several degrees of freedom become very complex and nonlinear. Working out the reverse solutions of these equations to deduce the necessary torques from the intended endpoint motion can be very time-consuming even for a large mainframe computer, and the obvious fact that the nervous system could not possibly frame the problem in terms of differential equations as we do does not change its computational complexity. How, then, can we make sense of the fact that biological solutions seem to be found almost effortlessly and in such a short time? A number of hypotheses have been put forward to explain this apparent paradox (cf. Schmidt, 1988; Whiting, 1984). We discuss only the conceptual approach that is most relevant to our work.

The basic idea was introduced more than 20 years ago in three seminal articles (Asatryan & Feldman, 1965; Feldman, 1966a, 1966b) which emphasized that any given position of a limb can be maintained by balancing the agonist and antagonist torques by setting the muscle stiffness appropriately. After noting that if the stiffnesses are changed the equilibrium of the torques is attained for a different set of muscles lengths, the authors suggested that a limb can be driven from one initial position to a specified target simply by modifying the stiffness of the muscles that control the joints. The interest of this idea vis à vis the computational complexity of motor planning is that only the essential features of the movement are supposed to be specified centrally, whereas the details (both geometric and kinematic) result from the intrinsic properties of the viscoelastic system formed by the muscles and the moving masses. Although the idea received some support from human (Abend, Bizzi, & Morasso, 1982) and animal (Polit & Bizzi, 1978) experiments that involved simple rotations of the forearm, it was soon clear that it is difficult to represent complex movements with sequences of point-to-point displacements. Moreover, neither the trajectories nor the kinematics of movements driven only by shifts in the equilibrium point mimic the actual observations very satisfactorily. Better results can be obtained with modified versions of the same basic idea (Berkinblit, Feldman, & Fukson, 1986; Feldman, 1974, 1986; Hasan & Enoka, 1985; Hogan, 1985). The original hypothesis of an abrupt transition from one equilibrium point to another is replaced with the more sensible notion of a continuous evolution of the equilibrium point along a virtual (internal) trajectory. Of course, the computational burden for the definition of the virtual trajectory increases considerably. Nevertheless, the motor system might use one of the aforementioned cost-minimization approaches to specify the virtual rather than the endpoint trajectory (Flash, 1987). Notice that the virtual trajectory may be less smooth and continuous than the actual one. In fact, as already stressed by several authors (e.g., Denier

van der Gon, Thuring, & Strakee, 1962; McDonald, 1966), the conversion of the efferent motor commands into forces can be described as an integration (i.e., a low-pass filtering operation) that has a smoothing effect.

The Oscillatory Hypothesis

Hollerbach (1981) presented behavioral evidence that even a movement as complex as writing can be construed as the result of modulating a basic sustained oscillatory mode (not necessarily a harmonic mode). Similar ideas had been expressed in describing finger and hand movements (Denier van der Gon & Thuring, 1965). Furthermore, quasiharmonic oscillations arise naturally when one describes the biomechanical and physiological properties of the muscles and the attached masses by using second-order differential equations (cf. Haken, 1977). This may suggest that the oscillatory mode is a universal characteristic of many human movements and that different gestures are obtained by efferent modulations of the amplitude, phase, and frequency parameters of this basic mode. In particular, the oscillatory hypothesis affords a very simple way of distinguishing the control parameters responsible for setting the size and shape of the trajectory (i.e., the amplitude of the oscillatory components) from those responsible for the kinematics (i.e., the frequency of the components).

Interestingly, most attempts to address either the degrees-of-freedom problem or the computational complexity problem discussed before also (implicitly or explicitly) conclude that hand motions ought to exhibit some of the characteristic features of (possibly damped) elastic oscillations. The case of elliptic trajectories is particularly interesting because many portions of hand trajectories can be approximated fairly accurately by elliptic segments; a complex movement may be seen as a splined sequence of such segments (Morasso, 1986). When the trajectory is an ellipse, the oscillatory hypothesis implies the possibility of approximating reasonably well the Cartesian components of the motion, either at the endpoint or at some intermediate link, by using segments of harmonic functions. Soechting et al. (1986) stated such a possibility explicitly and tested it experimentally in the case of two- and three-dimensional continuous movements. Moreover, both the equilibrium-point and the virtual-trajectory hypotheses make the same conclusion because of the nature of the mechanisms that are supposed to maintain static and dynamic equilibrium. In addition, our bottom-up approach leading to the two-thirds power law and isochrony (see the previous discussion) suggests a special connection between harmonic functions and a certain class of natural movements. Indeed, we demonstrate that if tangential velocity and radius of curvature are functionally related through the equation $V(t) = K R(t)^{1/3}$, and if the trajectory of the hand is an ellipse or a combination of ellipses, then the horizontal and vertical components of the movement are necessarily harmonic functions of equal frequency (Viviani & Cenzato, 1985; see also Appendix). The minimum-jerk model (Flash & Hogan, 1985; Hogan, 1984) predicts that the x and y components are quintic functions of time, but at least in the case of elliptic trajectories the coefficients of the polynomials are such that the difference

with respect to sine and cosine functions are undetectable experimentally. Some other models instead predict measurable, systematic departures from the ideal sinusoidal motion. A modified version of the Hogan and Flash idea—the viscoelastic model for jerk minimization (Wann et al., 1988)—allows temporal asymmetries in the x and y velocity components. Similarly, Maarse and Thomassen (1987) suggested that handwriting movements are best simulated by assuming time-asymmetric velocity profiles. Finally, when certain types of nonlinearities are introduced in the basic mass-spring model to account for stable limit cycles (Kay, Kelso, Saltzman, & Schöner, 1987), the motion is no longer a perfect sine wave.

Two things need to be stressed concerning the way elliptic motions are generated. First, that the components of the movements are approximately harmonic functions is not a trivial mathematical fact. In principle, infinitely many pairs of components—some of them sharply different from sine and cosine functions—could be used to trace the same ellipse. The particular solution that involves purely harmonic functions is a special (and somewhat ideal) case. With reference to the well-known patterns studied by the 19th-century French physicist Jules Lissajous, in this article we use the term Lissajous elliptic motion (LEM) to indicate such a special case. Second, we have seen how different conceptual frameworks may lead to the same general oscillatory hypothesis. Assessing their relative merits is almost impossible as long as the motion under examination is indistinguishably close to an LEM. Nevertheless, we have seen that some of these frameworks also make specific predictions about the possible deviations from the ideal sinusoidal case. Thus, it is important to investigate precisely those conditions in which systematic deviations can be measured.

The Developmental Approach

The main goal of the experiments reported in this article is to further the investigation of the constraints between trajectory and kinematics, which (as argued before) provide a clue to both the degrees-of-freedom problem and the computational complexity problem. The preceding arguments led us to believe that one way of pursuing this goal is to try to identify the more adequate conceptual scheme to account for the oscillatory behavior of hand movements that is suggested by so many converging lines of evidence. Our strategy is two-pronged: On the one hand, we accept the premise that an adequate identification is only possible by testing a candidate scheme across a range of different experimental paradigms that are likely to induce deviations from the sinusoidal model. On the other hand, because it is likely that such deviations can indeed be observed in the course of development (see the *Results* section), we select age differences as the main factor in our experimental paradigm. We consider the simplest task—the continuous drawing of ellipses—for which there is evidence of systematic departures from the simple sinusoidal model even in adults (Wann et al., 1988), and we investigate the differences between adult performances and the performances of children in the age range in which most children attain proficiency in manual skills (writing, drawing, and

playing musical instruments). We expect this developmental comparison to be useful for understanding the relationship between geometry and kinematics for the following reason. It is known that at 5 years of age children can already produce regular, smooth sinusoidal forearm movements similar to the components of an LEM (Mounoud, Viviani, Hauert, & Guyon, 1985; Viviani & Zanone, 1988). Thus, if the Lissajous mode of trajectory formation were an emerging property of the implementation stage specifically related to the active and passive properties of the biomechanical system and requiring no independent coordinative control of the components, there should be no systematic difference between children and adults in their drawing of an ellipse. In practice, however, several differences have been documented between the way children and adults execute the same movements of the upper limbs (cf. Wade & Whiting, 1986). Some of them are credited to a corresponding difference in the central representation of the motor plan (Mounoud, 1986; Mounoud et al., 1985; Sciaky, Lacquaniti, Terzuolo, & Soechting, 1987); others are credited to a different use of the proprioceptive and exteroceptive afferences for controlling the ongoing movement (Hay, 1979, 1981; Søvik, 1981; Von Hofsten, 1979). Developmental redistribution of the degrees of freedom along the biomechanical chain has also been cited as a source of age-related differences (Ziviani, 1983). In any case, there are reasons to suspect that children's execution of our drawing task may differ significantly from that of the control adult population and that the difference concerns the motor-control mode. If so, we could contrast a number of hypotheses on the nature of the difference; more specifically, we could test whether the two-thirds power law can be generalized to encompass both the fully mature performance and the preceding stages of motor development.

Our experiments were also designed to document the evolution with age of isochrony. It has been argued (Lacquaniti et al., 1984) that this phenomenon results from an automatic regulation of the average velocity as a function of the estimated linear extent of the trajectory to be executed. By necessity, this regulation must take place before the inception of the movement, in the preparatory stage of motor planning. Thus, the extent to which the degree of isochrony evolves with age ought to clarify whether this aspect of motor preparation is inborn or the result of motor learning. We also attempt to generalize the validity of the relationship between isochrony and the two-thirds power law that previous studies have suggested.

Method

Subjects

Six adults (4 men and 2 women, 26–44 years old) and 48 male Genevan primary-school children participated in the experiment on a voluntary basis. Children were divided into eight age groups. The mean (years and months) and the standard deviation (months) of the age in each group of 6 children were the following: Group 1, $M = 5.0$, $SD = 2.2$; Group 2, $M = 6.0$, $SD = 0.4$; Group 3, $M = 7.0$, $SD = 1.0$; Group 4, $M = 8.0$, $SD = 0.8$; Group 5, $M = 9.0$, $SD = 0.8$; Group 6, $M = 10.0$, $SD = 1.6$; Group 7, $M = 11.01$, $SD = 1.9$; Group

8, $M = 11.11$, $SD = 1.9$. Experiments were conducted at the earliest 2 months before a subject's birthday and at the latest 2 months after. All subjects were right-handed and had normal or corrected-to-normal vision.

Apparatus

Drawing movements were recorded with a Calcomp Series 9000 digitizing table (California Computer Products, Inc., Anaheim, CA) (sampling rate: 88 Hz; nominal accuracy: 0.01 mm; usable workspace: 90 × 90 cm) connected to a personal computer. The writing implement was the standard pen supplied with the table: It resembles a normal ballpoint pen and leaves an analogous visible mark on paper. The table was mounted on a support frame that permitted the experimenter to adjust the inclination of the workplane according to each subject's preference. The inclination never exceeded 20°. Subjects were comfortably seated and were allowed to lean and support themselves on the table. Experiments were run in a quiet room of the primary school that all of the children were attending. Templates for the movements to be executed were drawn on a regular A3-sized paper sheet placed on the table surface. The templates consisted of 10 elliptical outlines arranged spatially as shown in Panel A of Figure 1. The eccentricity of the ellipses was $\Sigma = 0.9$. Their perimeter P varied between 2.34 cm and 53.02 cm in a logarithmic sequence ($P_1 = 2.34$ cm; $P_2 = 3.31$ cm; $P_3 = 4.69$ cm; $P_4 = 6.63$ cm; $P_5 = 9.37$ cm; $P_6 = 13.26$ cm; $P_7 = 18.75$ cm; $P_8 = 26.51$ cm; $P_9 = 37.49$ cm; $P_{10} = 53.02$ cm). In all cases the major axis of the ellipses was rotated by 45° with respect to the horizontal. (Previous studies with adults and pilot experiments with children have shown that the posture required by this orientation is the most comfortable one for executing free movements.)

Task

The task consisted of tracing each elliptical outline freely and continuously for about 14 s. Subjects were left free to choose the rhythm of movement but were instructed to try to maintain a constant rhythm throughout all movement cycles for any given ellipse. Movements were recorded during a period of 10 s. The experimenter started the recording after a few cycles of movement during which subjects reached a stable pace. Because the pen left a visible mark, the template outline was no longer clearly visible by the time the recording started: Subjects were guided visually by the traces they had left on paper during the warm-up cycles.

Procedure

Each subject participated in a single experimental session in which the entire sequence of 10 ellipses was traced twice, with a short period of rest between trials and a longer period between sequences. A trial could be repeated if for any reason the subject was dissatisfied with the performance. A new sheet with the template outlines was provided for each sequence and whenever a trial was repeated. In half of the subjects, the orientation of the outlines for the first sequence was the one shown in Panel A of Figure 1; in the other half, the sheet was placed upside down. In all cases, the orientation was inverted for the second sequence. The session began with an introductory phase in which the experimenter explained the task and demonstrated the use of the pen. A few pretest trials were administered to familiarize the subjects with the equipment and to let them find the most comfortable posture. The order in which the 10 ellipses had to be traced was given by the experimenter and was different for the two sequences in a session. For each subject, the orders were selected at random (without

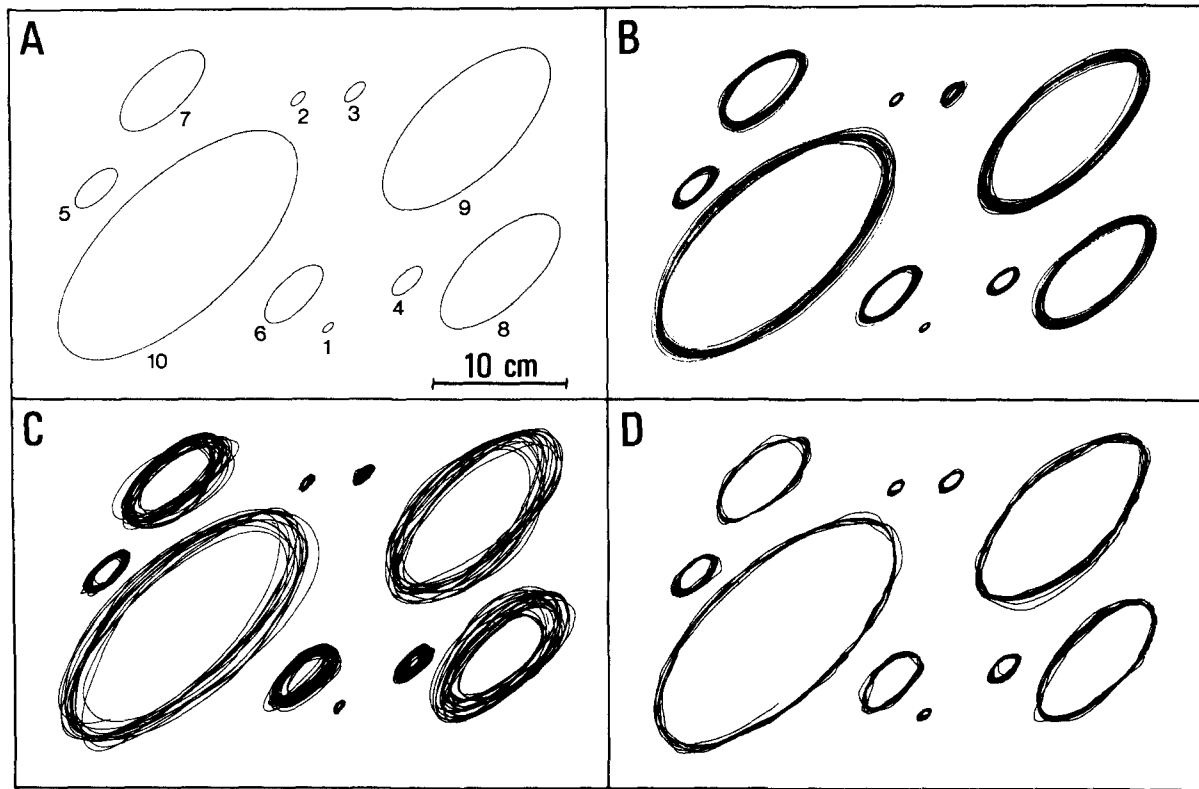


Figure 1. Movement templates and typical trials. (Panel A depicts the complete set of 10 elliptic templates as they were arranged spatially within a regular A3-sized paper sheet. All templates were traced successively in random order. In one series of recordings the orientation of the sheet was the one shown. In a second series the sheet was turned upside down. The other three panels illustrate one complete series of trajectories produced by an adult [Panel B], a 7-year-old child [Panel C], and a 10-year-old child [Panel D]. The results in Panels C and D illustrate two of the most typical styles of performance observed in children.)

replacement) from a subset of all possible permutations of the first 10 digits.

Results

Styles of Performance

All subjects completed the two series of recordings without difficulty. We did not search for statistical evidence of motor learning trends; however, no qualitative difference was detected between the performances at the beginning and at the end of a session. The two most obvious discriminating factors among children of different ages and between children and adults were (a) the degree of accuracy with which the shape of the templates was reproduced and (b) the cycle-by-cycle consistency. At all ages performers idiosyncratically chose a baseline tempo and spontaneously tended to maintain this tempo throughout. Generally, both accuracy and consistency increased with age, but large individual differences emerged even within age groups. These differences were partly the consequence of the style of motor performance adopted by the subjects. Panel B of Figure 1 shows the performance of a 42-year-old subject who exhibited the accuracy and fluency

of execution that is typical of most adults. Panel C illustrates the performance of a 7-year-old child. His style of performance—characterized by a fast tempo, good fluency, and a considerable amount of geometric variability—was not uncommon among both the youngest and oldest children. Panel D illustrates another typical style observed most often in the intermediate age groups. The results are from a 10-year-old boy who was clearly very concerned about the spatial accuracy of the traces. Movement runs less freely than in the examples of Panels B and C; the general tempo of the movements is much slower, and the traces are not as smooth. Whereas the child from Panel C seems to apply only a global control over the gesture (as adults generally do) and is able to plan an entire cycle, the child from Panel D clearly exerts continuous local visual control on the trace. Each cycle of this boy's performance results from a sequence of smaller units of actions.

Geometric Parameters

For each trial several geometric parameters of the traces were measured by off-line processing of the position data. We used the following procedure to determine the best elliptical

approximation to the entire trajectory. First, we computed the center of gravity of the trajectory by averaging separately the horizontal and vertical coordinates of the sampled points of the trace. Then, after considering the samples as pointlike unitary masses, we computed the central moments of inertia of the trajectories for all axes that passed through the center of gravity (*inertial tensor*). It is known (Goldstein, 1980) that the inertial tensor of a two-dimensional distribution of pointlike masses can always be represented by an ellipse. Moreover, we can show that if the traces were perfectly elliptical the directions of the largest and smallest eigenvectors of the inertial tensor (i.e., the axes around which the moment of inertia of the masses is maximum and minimum) would coincide with those of the minor and major axes of the traces, respectively. Their amplitude would be scaled by a factor $\sqrt{2}$ with respect to the corresponding axis. Real traces are both distorted and different from cycle to cycle. Nevertheless, the inertial tensor ellipse rotated by 90° and scaled by $\sqrt{2}$ still provides the best least squares approximation to the trajectory of the movement. By this procedure we estimated three parameters that measure the global geometric accuracy of the performances: (a) the ratio between the perimeter of the best fitting ellipse (P_0) and that of the template (P_T), (b) the aspect ratio B/A between the minor and major axes of the best fitting ellipse, and (c) the inclination of the major axis of the trajectory with respect to the horizontal. Averages of the first two parameters over all subjects in each age group and both sequences of trials are shown in the two left panels of Figure 2. The corresponding averages for the inclination of the major axis are shown in the upper-right panel of Figure 2. The results indicate a monotonic age-related trend in all three parameters, with younger children tending to produce larger, less eccentric ellipses than the models (notice that adults' traces are slightly but consistently more eccentric than the templates). Younger children also tended to rotate the major axis of the trace toward the vertical.

The last geometric performance descriptor was the variability of the traces: this parameter was defined as follows. Consider a point (sample) of the trajectory and the line that joins the point to the center of gravity of the traces. We define the spatial error associated with the direction of this line as the distance between the sample and the intercept of the line with the best fitting ellipse. Conventionally, a positive value is assigned to the error if the sample lays outside the best fitting ellipse and a negative value is assigned if it lays inside. As expected, the average absolute error e_a over all points of a trace increased with the perimeter of the template. For each subject, however, the scaling of the average error with P_T could be represented accurately by an empirical power function $e_a = e_0 P_T^\delta$. Thus, a relative error e_r for each point of the trajectory was obtained by normalizing the corresponding absolute error with the empirical function $e_r = e_a / P_T^\delta$. Finally, we divided the space into 360 one-degree angular sectors that originated from the center of gravity of the ellipse, and we defined the average trace variability within each sector as the standard deviation of the normalized error for all points of the trace that were in that sector. We computed a polar plot of the spatial variability for each age group by pooling the data for all subjects within the group, all template sizes, and

both sequences. Figure 3 illustrates the results of this procedure for the four indicated ages. For graphical convenience, the ellipses have been rotated clockwise by 45° . These plots show that geometric variability was not uniform for all directions: It was systematically smaller when the curvature was higher. A similar anisotropy was also present in the groups of children not shown in Figure 3, as well as in the adult group. We obtained a global measure of the individual spatial accuracy by averaging the corresponding radial variability over all directions. Means and 95% confidence intervals of the individual averages for all age groups are shown in the lower-right panel of Figure 2. The results confirm the trend of the P_0/P_T plot in the same figure: The youngest subjects (5-year-olds) stand out as being much less consistent than all other children. Moreover, the variability of the traces for the children as a whole was more than twice as large as that for adults. Even the oldest children (12-year-olds) had not yet reached a fully mature performance.

Temporal Parameters

There was considerable variability in the general tempos the subjects chose for tracing the ellipses. Within each series of recordings, however, the average velocity covaried with template size in a strikingly consistent manner. It is well documented (see the introduction) that in adults the average velocity of planar movements increases with the linear extent of the trajectory (isochrony). In the case of periodic movements along a two-dimensional trajectory, this phenomenon is best described as a relationship between the period T of the movement and the perimeter P of the trajectory being traced. In Figure 4 different symbols describe this relationship for each age group. The ordinates of the data points are the averages over all complete cycles of movement of the time necessary to complete one cycle. The abscissas are the average linear extents of the cycles. We pooled data from the two sequences of recordings and all subjects in each age group. In the double-logarithmic scale used for this plot, T is nearly a linear function of P at all ages. Thus, the power function shown inset is an accurate (albeit empirical) representation of the relationship between perimeter and period. A linear regression analysis of the $\log P$ - $\log T$ relationship was performed separately on each cluster of 120 ($6 \times 2 \times 10$) data points in relation to a group of subjects (Figure 5). Despite the variability of the general tempo chosen by each subject, the coefficients of linear correlation (open triangles) confirm the validity of the power-law representation for each age group. We estimated the exponent of the power law (solid circle) by the slope of the linear regression of $\log T$ over $\log P$. The value of γ is almost constant for all children and drops somewhat in adults. The intercept parameter of the regression ($\log T_0$) decreases with age in the children and increases in adults.

These data quantitatively describe the form that isochrony takes in our experimental conditions. The quantity $1/T_0$ represents a baseline tempo selected idiosyncratically by each subject and spontaneously kept constant throughout (the instructions only emphasized the request to maintain the same rhythm in each trial). The actual rhythm of the move-

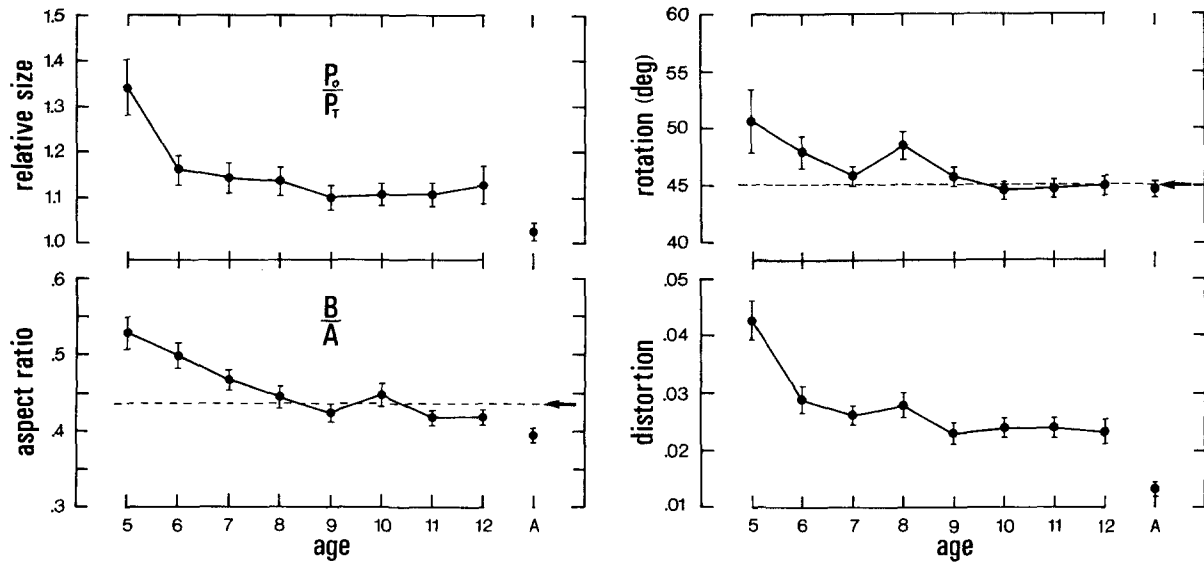


Figure 2. Geometric parameters as a function of age. (The upper-left panel denotes the ratio between the average perimeter of the trace [P_0] and the perimeter of the template [P_t]. The lower-left panel denotes the ratio between the minor [B] and major [A] axes of the best elliptic approximation to the traces [A on the x axis represents adults]. The dashed line indicates the ratio [.437] that corresponds to the eccentricity [$\Sigma = 0.9$] of the templates. The upper-right panel denotes the inclination of the major axis of the best elliptic approximation to the trace. The lower-right panel denotes the variability of the trace as measured by the standard deviation of the radial error with respect to the elliptic approximation [A = adults]. Errors are normalized to the perimeter of the template. In all cases data points are averages over all trials of all subjects. Bars indicate the 95% confidence intervals of the means.)

ment ($1/T$) for each template results from scaling this base value with the perimeter of the intended trajectory. The exponent γ measures the strength of the scaling. In the limit case $\gamma = 0$, the rhythm is constant for all perimeters (perfect isochrony). By contrast, $\gamma = 1$ implies that T is proportional to P , that the average tangential velocity is constant for all templates, and that there is no compensation. Even though on average adults were closer to isochrony than children, the data in the upper panel of Figure 5 show conclusively that this compensatory mechanism is already in place at the age of 5 years.

It has been suggested (Wann & Jones, 1986) that some writing difficulties experienced by 9- and 10-year-old children correlate with a lack of stability in the temporal structure of the writing movements. We explored the stability issue by computing for each trial the coefficient of variability (standard deviation/mean) of the period T . The lower panel of Figure 5 shows the means and the 95% confidence intervals of this coefficient for all 120 trials recorded for each age group. Children were more than twice as variable as adults, but we found a clear age-related trend in the cycle-by-cycle temporal stability of the movement; however, the improvement with age is slow and not monotonic. In particular, there is evidence of a regressive phase in 7-year-old children, whose timing stability drops by almost 50% with respect to the preceding age group. Notice that this deterioration partly reflects the emergence of the discontinuous style of execution documented in Panel D of Figure 1.

Relationship Between Geometry and Kinematics

In adults, when the trajectory of a planar movement has no points of inflection, the two-thirds power law applies throughout the movement. Figure 6 illustrates this with the data in relation to two ellipses (P_4 and P_8) traced by 1 adult subject. As in Figure 4, a linear relation in doubly logarithmic scales corresponds to the expression $V = K R^\beta$ shown inset. The slope and the intercept parameter of the $\log V$ - $\log R$ linear regression can be used to estimate the exponent β and the multiplicative constant K of the power law, respectively. For obvious reasons, we call the constant K the *velocity-gain factor*. A regression analysis of $\log V(t)$ on $\log R(t)$ was performed separately for each trial of each subject in each age group. A two-way analysis of variance (ANOVA) (9 ages \times 10 perimeters, with 12 replicates per cell) demonstrated a significant effect of both variables of the experimental design (age and template size) on the velocity gain: for age, $F(990, 8) = 43.53$, $p \ll .0001$; for perimeter, $F(990, 9) = 81.51$, $p \ll .0001$. Interaction was nonsignificant, $F(990, 72) = 0.53$, $p = 1.000$. In addition, exponent β was found to depend on age, $F(990, 8) = 66.28$, $p \ll .0001$ and on the perimeter, $F(990, 9) = 18.74$, $p \ll .0001$, with no significant interaction, $F(990, 72) = 0.99$, $p = .512$. Nevertheless, the influence of the perimeter on β was actually limited to the children. By collapsing the data over all children groups, we demonstrated a weak but significant linear trend in the $\log P$ - β relationship: slope = -0.058 , intercept = 0.338 , correlation = $.370$, $p \ll$

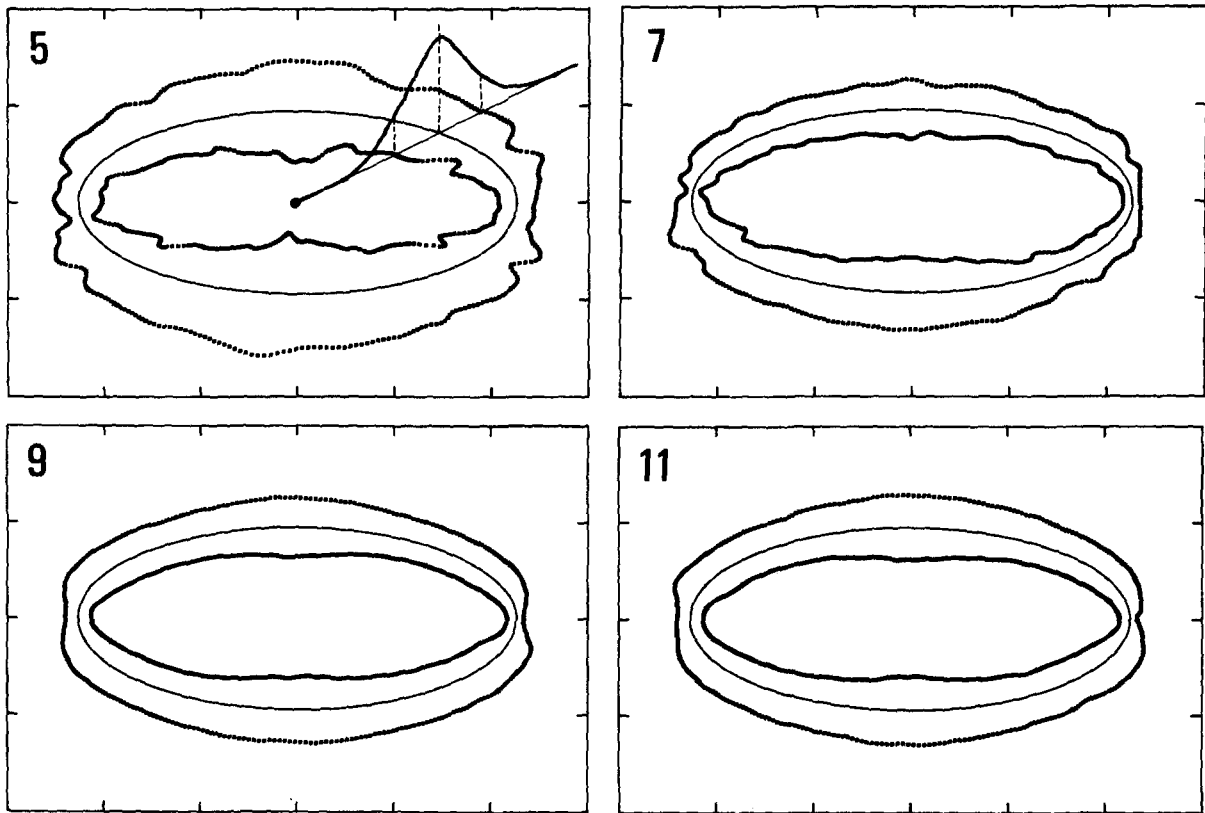


Figure 3. Geometric variability as a function of age. (In each panel the bands around the templates encompass ± 1 standard deviation of the absolute radial error with respect to the ideal trajectory. Data for all trials and all subjects in the indicated age groups have been pooled. The plot for adults is similar to that for 11-year-old children. Notice that the variability is not uniform around the trajectory.)

.0001. Instead, no trend existed in adults: slope = -0.002 , intercept = 0.347 , correlation = $.070$, $p = .443$. The effect of age on the exponent of the power law can be summarized by collapsing the values of β over all perimeters (Figure 7). The within-group average of the coefficient of linear correlation (upper panel) exceeds $.95$ for all but the youngest children. Regardless of age, then, in children the relationship between tangential velocity and radius of curvature can be described accurately by a power law. As expected from the results of previous studies, the average of β for the adult group was close to one third: The performance of adults agrees with the two-thirds power law (see the *Internal Consistency* section). By contrast, the averages for all child groups were significantly smaller. Except for 5-year-old children, the exponent increased consistently with age, but as in the case of the geometric parameters, even the oldest children differed significantly from the adults.

Within a sequence of trials the velocity-gain factor K varied with the perimeter P . In Figure 8 different symbols illustrate this relationship for each age group. As in Figure 4, the abscissas of the data points are the linear extent of the movement cycles averaged over all trials and all subjects within the group. The ordinates are the corresponding averages of K .

Here again, linear functions describe fairly accurately the relationship between the logarithms of the two variables. Thus, for each group of subjects, the velocity-gain factor can be described empirically by a two-parameter power function of the perimeter. In addition, the interpretation of the results in Figure 8 is similar to that for the timing data. We assume that the velocity-gain factor is set by modulating a baseline gain K_0 by the perimeter of the intended movement. The baseline value is chosen idiosyncratically by each subject, whereas the modulating effect of the movement size is taken to represent a built-in property of the motor-control system that is common to all subjects. Later, we present a derivation of the timing data of Figure 5 from the results illustrated in Figures 6 and 8.

We estimated the parameters K_0 and α with a linear regression analysis of $\log K$ on $\log P$ separately for each subject (data for the two sequences of trials were pooled). A one-way ANOVA (9 ages, with 6 replicates per cell) demonstrated a significant effect of age on both the multiplicative constant K_0 , $F(45, 8) = 4.17$, $p = .001$, and the exponent, $F(45, 8) = 2.16$, $p = .05$. Accordingly, we averaged the two parameters of the regression and the corresponding coefficients of correlation over all subjects within each age group. The results are

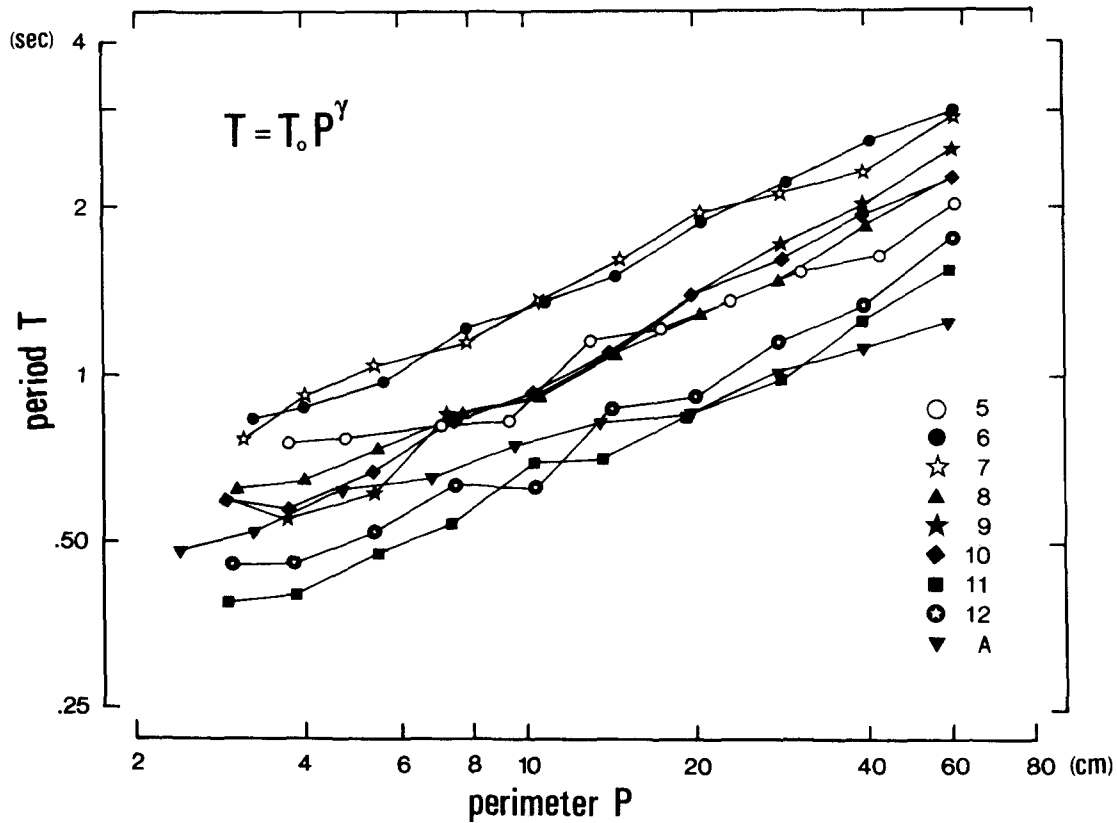


Figure 4. Isochrony: The relationship between the perimeter P of the trace and the duration T of a movement cycle. (We obtained each data point by first computing the mean perimeter and the mean period for each trial and each subject. Then we averaged the individual means over all trials and all subjects [12 measures]. Different symbols identify the age groups. In a double-logarithmic scale all T - P relationships are well approximated by linear functions. Thus, in linear scales, P and T are related by the power function $T = T_0 P^\gamma$. The intercept of the linear regression with the line $P = 1$ estimates $\log T_0$. The exponent γ is estimated by the slope of the linear regression. Both of these parameters vary with age [see Figure 5].)

shown in Figure 9. The behavior of the adult subjects confirmed the conclusion of previous experiments: The velocity-gain factor increases faster than the cubic root of the perimeter. The data from the children were qualitatively similar. The two parameters of the power law vary across ages, however: The baseline gain K_0 tends to increase, whereas the strength of the perimeter modulation drops by almost 50% between ages 5 and 12. Strangely enough, the behavior of children diverges progressively from that of adults as their age increases.

Internal Consistency

In the introduction we mentioned that if the exponent β in the relationship $V(t) = K R(t)^\beta$ is equal to $1/3$, then it follows necessarily that all elliptic movements are LEMs. The adult data presented here are hardly new: They simply confirm the results of previous studies (Lacquaniti et al., 1983) by showing that in these subjects the condition $\beta = 1/3$ is satisfied almost perfectly. Therefore, when adults trace either isolated ellipses

or more complex figures composed of ellipses, the components of the movements are indeed harmonic functions of equal frequency. By contrast, the finding that children systematically violate the condition $\beta = 1/3$ is new and has a pivotal importance for the logic of this article: We must provide additional evidence that the violation is real. In particular, one has to rule out the possibility that values of β different from $1/3$ are an artifactual result of computing a linear regression analysis on nonlinear data. To this end, we performed three further analyses of the data. First, with a different data analysis we checked the statistical significance of the β estimates in adults. The parameters of the linear correlation between $\log R$ and $\log V$ calculated before with all data points could be biased because successive samples of a continuous trajectory are not independent (Morrison, 1976). This potential source of bias can be drastically reduced by considering a regularly interspersed subset of the 880 samples available for each trial. In particular, we calculated the linear regression parameters again for each trail by selecting only 1 point in 5. For each template size, Table 1 reports the new β estimate

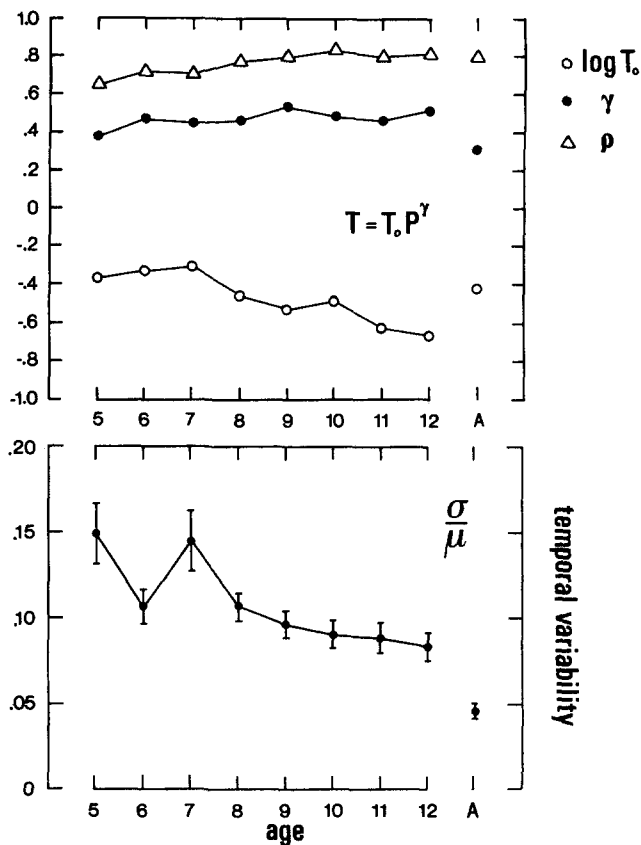


Figure 5. Linear regression analysis of the log T-log P relationship. (The upper panel shows isochrony parameters as a function of age. Circles represent the intercept [open circles] and the slope [solid circles] of the linear regression between log T and log P [see Figure 4]. Triangles represent the corresponding coefficients of linear correlation. The regression analysis was performed on the averages for individual trials. The results indicate that almost irrespective of age, the duration of one cycle of movement increased more slowly than the square root of the perimeter. Notice that the baseline tempo $1/T_0$ increased with age in the child population. Adults did not follow this trend. The lower panel shows temporal variability as a function of age. Data points represent the average over all trials and all subjects [120 measures] of the coefficient of variability of the movement period. Bars encompass the 95% confidence interval of the mean. As judged by this parameter, timing in adults is at least twice as stable as in the child group. Seven-year-old children do not fit the general trend of improving stability with increasing age.)

averaged over all subjects and the two repetitions. The associated 95% confidence interval is the largest among the 12 (6 subjects \times 2 repetitions) intervals calculated for individual trials. We averaged the coefficients of linear correlation by using Fisher's hyperbolic tangent transform (Kendall & Stuart, 1979). The results agree with the global analysis summarized by the two data points in Figure 7 that are relative to the adult control group.

The other two analyses were based on a generalized version of the two-thirds power law. The formal developments that led to the predictions actually tested are fully described in the Appendix. Here we present the qualitative arguments. Con-

sider the general class of elliptic motions whose tangential velocity satisfy the constraint $V(t) = K R(t)^\beta$ for some positive values of K and β . We use the term generalized Lissajous elliptic motion (GLEM) to define this class. LEMs such as those produced by our adult subjects are simply special cases of GLEMs corresponding to the condition $\beta = 1/3$. We can show that for a given ellipse (i.e., for fixed values of the perimeter P and the eccentricity Σ) and for a given value of the velocity-gain factor K, the period T of a GLEM is uniquely defined by the exponent β (see Equation 13 in the Appendix). Thus, a comparison between the experimental (T_e) and theoretical (T_i) values of the period affords a stringent test of the internal consistency of the results. Internal consistency implies that the observed deviations of the exponent from the value $1/3$ are not artifactual. The comparison was performed on all 1,080 recorded trials. We calculated the perimeter and the eccentricity of each trace from their best elliptical approximation (see the previous discussion). The average gain K and the exponent β were estimated from the regression of $\log V$ on $\log R$ (as illustrated in Figure 6). The 1,080 pairs of T values (observed and predicted) were binned into adjacent classes according to the corresponding value of β (class size = 0.1). Finally, within each class separately we performed a linear regression analysis of the two measures of the period to verify that high correlation is not the artifactual result of increasing the range of variability of the variables being regressed. The results (Figure 10) are very satisfactory. Indeed, independent of the preceding formal developments, the vari-

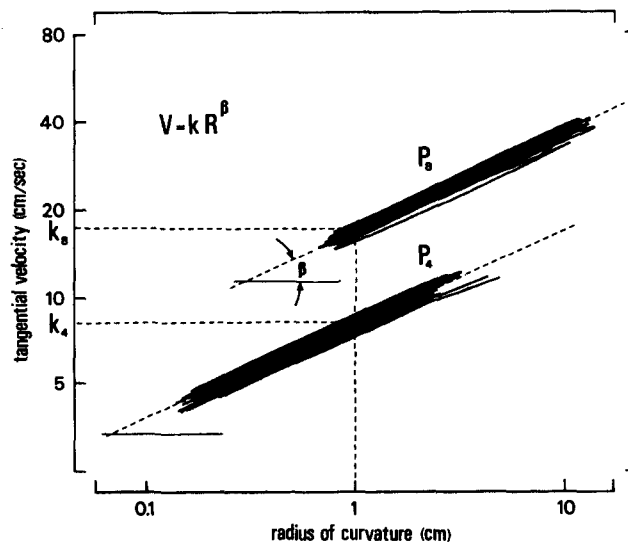


Figure 6. The two-thirds power law: data from two trials (templates P_4 and P_8) in 1 adult subject. (These typical examples demonstrate the linear relationship that exists between the logarithms of the radius of curvature and the tangential velocity. In linear scales, this relation corresponds to the power law $V = K R^\beta$. The constant K [velocity-gain factor] is estimated by the intercept of the linear regression with the line $R = 1$. The exponent β is estimated by the slope of the regression line. In adults, the exponent is insensitive to the size of the movement and takes values close to $1/3$. Changes of the average velocity as a function of size [see Figures 3 and 4] are only reflected in a modulation of the velocity-gain factor K.)

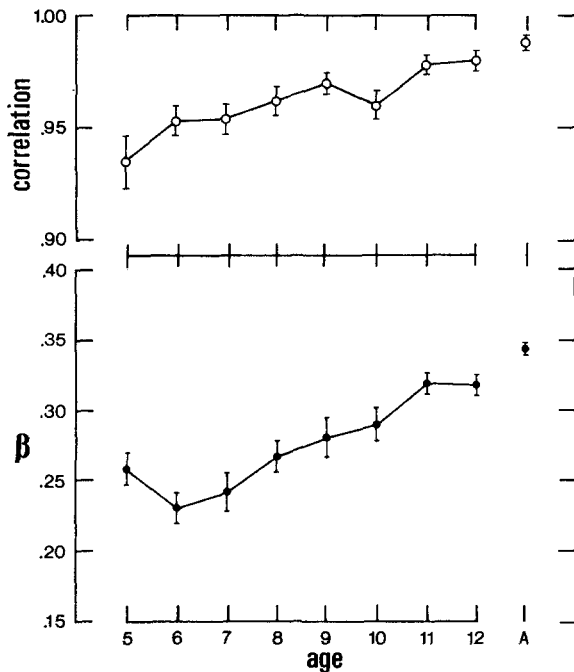


Figure 7. Evolution with age of the parameters of the two-thirds power law: exponent. (The lower panel denotes exponent β in the empirical power function $V = K R^\beta$. β was estimated by linear regression analysis [as illustrated in Figure 6]. The upper plot denotes the coefficient of linear correlation for the log R -log V relationship. In both plots data points are averages over all trials and all subjects in each age group [120 measures]. Bars encompass the 95% confidence interval of the mean. At all ages, the exponent in children differs significantly from the typical adult value. Convergence toward this value is not achieved yet at 12 years of age. Five-year-old children do not fit this age-related trend.)

ability of the actual periods within a class can always be considered the result of the combined variability of two factors: the perimeter of the trace and the average velocity of the movement. The fact that the periods are accurately predicted by a formula that contains the parameters K and β explicitly and that these parameters were computed under the assumption that the movements are GLEMs strongly confirms the validity of the assumption.

We carried out a third test of the fact that the deviations of the exponent β in children's movements is not artifactual by comparing the theoretical and experimental time courses of the velocity components. We can show that the horizontal ($x(t)$) and vertical ($y(t)$) components of a GLEM are solutions of a pair of nonlinear homogeneous differential equations (see Equations 9 and 10 in the Appendix). For a given shape and size of the ellipse and for any choice of the parameters K and β , we can predict both the displacement components of the corresponding GLEM and the associated velocity components $dx(t)/dt$ and $dy(t)/dt$. Panels A and B of Figure 11 show two complete cycles of the theoretical velocity components for 13 equally spaced values of β between 0 and 1. For this simulation, the velocity-gain factor K and the perimeter P have been fixed to 10 and 26.51 (P_8), respectively. The eccentricity was

the same as that of the templates ($\Sigma = 0.9$). To make visualizing the difference between horizontal and vertical components clear, the simulation is relative to an ellipse whose major axis is horizontal. Moreover, to highlight the effect of changing the exponent, we scaled the time axis for each value of β by the corresponding value of the period T (see the previous discussion and Figure A1 in the Appendix). Units on the vertical axes are arbitrary. Clearly, the velocity (and therefore the displacement) components of a GLEM are not in general harmonic functions. They are strictly time-symmetric, however. Panels C and D in Figure 11 show eight examples of velocity components from children's recordings. To facilitate the comparison with the theoretical predictions, we selected the examples among the trials with the smallest values of β (see Figure 11). We computed each velocity profile from the displacement data after rotating the corresponding original trajectory clockwise by an amount equal to the inclination of its major axis. The deviations of the experimental velocity components from sine and cosine functions are similar to those predicted by the GLEM model. The values of β have predictable effects on the law of motion, which supports the contention that GLEM provides a unifying framework for describing both adult and child motor performances.

Discussion

In the introduction we argued that an effective approach to both the degrees-of-freedom problem and the computational complexity problem is to investigate the constraints that limit the number of possible implementations of a given motor goal. The aim of the experiments reported here was to pursue this approach in the specific case of those constraints that manifest themselves as covariations between geometric and kinematic variables of the endpoint trajectory. The basic assumption in designing the experiments was that an insight into the nature of these constraints can be gained through the developmental approach, that is, by comparing the motor performances at various stages of childhood with mature adult behavior.

The results concern two aspects of arm-hand movements that are well documented in adults: (a) the scaling of endpoint velocity with movement size (isochrony) and (b) the dependence of velocity on the curvature of endpoint trajectory (two-thirds power law). Age-dependent differences have been demonstrated for both of these phenomena. The implications of the findings vis à vis the aim of the study are somewhat different, however, and need to be discussed separately. First, we consider the phenomenon of isochrony and then the velocity-curvature relationship. Finally, we attempt to outline a possible integration of these two aspects of the control of movement.

Isochrony

The notion of movement size that occurs in the definition of isochrony must be qualified as a function of the type of motor task that is executed. With simple closed trajectories

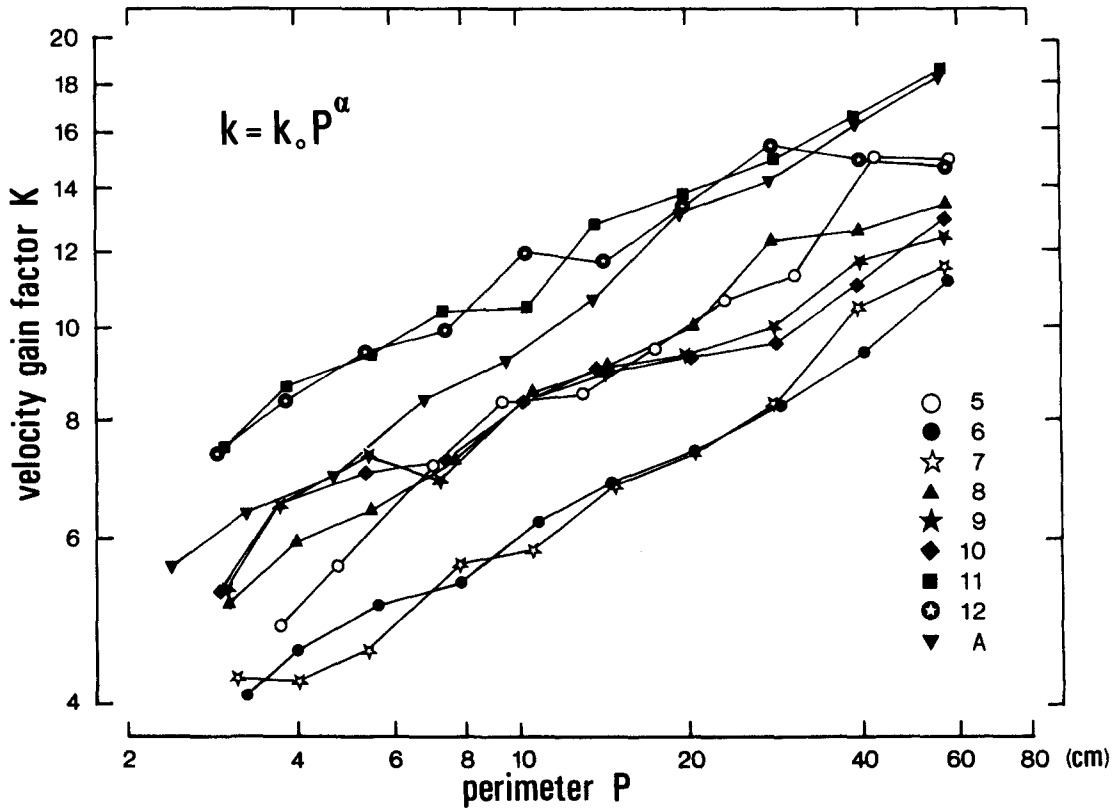


Figure 8. Evolution with age of the parameters of the two-thirds power law: velocity-gain factor. (Data points are averages over all subjects in each age group and all trials for a given template [12 measures]. Abscissas indicate the mean perimeter of the trace. Ordinates indicate the parameter K in the empirical power law $V = K R^\beta$. K was estimated by linear regression analysis [as illustrated in Figure 6]. In a doubly logarithmic scale all K-P relationships are well approximated by linear functions. Thus, in linear scales, K and P are related by the power function $K = K_0 P^\alpha$. The intercept of the linear regression with the line $P = 1$ [not visible in this plot] estimates $\log K_0$. The exponent α is estimated by the slope of the linear regression. Both of these parameters vary with age [see Figure 9].)

traced repeatedly and continuously, a movement aspect that is directly involved in setting the average velocity is the total perimeter P of one movement cycle (Lacquaniti et al., 1984; Viviani, 1986; Viviani & McCollum, 1983). In these cases isochrony manifests itself as a power-function relationship between perimeter and average velocity, or equivalently as a power-function relationship between perimeter and cycle period. This second mode of representation was adopted in Figure 4. Such an empirical fitting described the covariation between P and T with uniform accuracy for all age groups. Thus, the discussion of the results can be based exclusively on the two parameters T_0 and γ of the power function. We assume (see the Results section) that when faced with the sequence of templates of different sizes, subjects idiosyncratically select a baseline tempo ($1/T_0$) that is kept spontaneously constant throughout the execution of the series. We also assume that the average velocity in a cycle of movement ($V_m = P/T$) results from scaling this baseline tempo by a factor that depends on the estimate of the perimeter P. This modulating action, which represents the very essence of isochrony, presupposes that the motor-control stage responsible for set-

ting the velocity of the movement has access to an accurate quantitative estimate of the length of the intended trajectory. The stability of the P-T relationships (Figure 4) and the fact that the corresponding exponent γ is almost constant across all children's groups (Figure 5) indicate that (a) such an accurate estimate is already available before the age of 5, and (b) the mechanisms that set the velocity on the basis of this length estimate do not evolve between 5 and 12 years of age. Such a conclusion does not conflict with the fact (Figure 5) that age significantly affects the within-trial variability of the period. Indeed, we have seen (Figure 2, lower-right panel) that geometric variability also decreases with age and that the cycle-by-cycle temporal variations were mostly due to corresponding variations in the size of the trace especially in younger children.

The average adult values of the baseline tempo parameter and the exponent γ deviate significantly from the age trend within the child population. Adults were intrinsically faster than children (lower T_0), and they kept the variations of the period across the sequence of template sizes within a 2.5 to 1 ratio, which agrees with the results of Viviani and McCollum

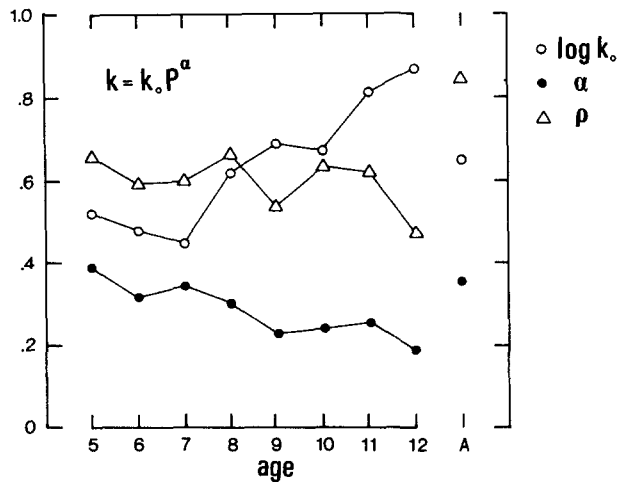


Figure 9. Evolution with age of the parameters of the two-thirds power law. (Circles represent the intercept [open circles] and the slope [solid circles] of the linear regression between $\log K$ and $\log P$ [see Figure 8]. Triangles represent the corresponding coefficients of linear correlation. Regression analysis was performed on the averages of K and P over the two trials for each subject and each template size. The regression parameters were then averaged over all subjects in each age group. Within the child group, the sensitivity of the velocity-gain factor to the movement size (α) decreased with age, whereas the baseline value K_0 increased. This reciprocal behavior is confirmed by the data points for the adult controls, which nevertheless do not fit the age-related trend exhibited by children.)

(1983). By contrast, children typically produced a 4 to 1 ratio. The faster movement rate in adults is in keeping with the generally increased fluency in most graphic skills that occurs with increasing age (e.g., Ziviani, 1984). The difference in the degree of isochrony can be ascribed either to the metric of the internal representation of the templates or to the mechanisms that translate the metric properties of the trajectory into an appropriate set of kinematic parameters. It is clear, however,

Table 1
Detailed Analysis of Adult Performances

P	av_{β}	SD_{β}	c_{max}	r
P1	.344	.017	.027	.989
P2	.341	.014	.023	.986
P3	.338	.014	.029	.987
P4	.335	.019	.025	.989
P5	.339	.017	.029	.989
P6	.339	.029	.024	.991
P7	.341	.010	.020	.992
P8	.340	.013	.020	.990
P9	.338	.013	.020	.991
P10	.334	.015	.022	.989
Total	.339	.016		.989

Note. For each adult trial ($N = 120$) we computed the exponent β , its 95% confidence interval, and the correlation r between $\log V$ and $\log R$. For each perimeter (12 trials) the average (av_{β}) and standard deviation (SD_{β}) of the exponent, the largest among the 12 95% confidence intervals (c_{max}), and the average coefficient of linear correlation (r) is indicated. (Fisher's z transform was applied before averaging correlations.) P denotes parameter.

that our experiments do not permit us to decide between the two hypotheses. Indeed, it is not even obvious whether any behavioral experiment may afford discriminatory evidence on this point. At any rate, regardless of the quantitative differences that exist between children and adults, the pattern of results suggests that the basis for isochrony is laid in the early stages of motor development (see the following discussion).

It might be interesting to investigate further the aforementioned assumption that a significant degree of isochrony necessarily implies the availability of some accurate estimate of the linear extent of the trajectory. In previous reports on adult subjects (Schneider, 1987; Viviani, 1986; Viviani & Cenzato, 1985), we presented evidence that complex movements are decomposed into units of motor action and that isochrony applies independently to each one of these units. If so, we ought to be able to use the degree of isochrony as a criterion for identifying the emergence of the simplest units of motor action in very young children and for studying the process by which several units are coupled in the planning of more complex movements. On this speculative note we conclude the discussion of isochrony; we now consider the second constraint observed in the execution of elliptic movements.

Relationship Between Curvature and Velocity: A Developmental View

First, we consider the question raised in the introduction: Are developmental data conducive to a better understanding of the relationship between radius of curvature and velocity? To discuss this we must reassess the current status of the empirical two-thirds power law, which purports to represent the final state of the developmental process. A recent study on adults (Wann et al., 1988) reported a replication of the experiments that were at the origin of the law (Lacquaniti et al., 1983; Viviani & Terzuolo, 1982) and questioned the heuristic value of the power-law formulation. The main points of the study by Wann et al. are the following: (a) When the eccentricity of the ellipse and the rhythm of the movement are both low, the exponent of the law, as measured by a global regression technique similar to that illustrated in Figure 4, deviates from the theoretical $1/3$ value. (b) This deviation can be interpreted as the result of pooling two heterogeneous sets of data points in the $\log R$ - $\log V$ diagram. Each set is well interpolated by a straight line, but the slopes of the lines are different. (c) Slope differences can be modeled by a viscoelastic generalization of the minimum-jerk model (mentioned in the introduction). (d) As the pace of the movement increases, slope differences disappear, and the original minimum-jerk model accurately predicts the two-thirds power law.

To explain point (b), Wann and his colleagues assumed that peak velocities for both horizontal and vertical components are not symmetrically placed within each half-cycle of the movement (cf. Wann et al., 1988, p. 627, Figure 2). Thus, they suggested that elliptic trajectories are not traced as a whole but rather as a sequence of four quarter-cycle segments each corresponding to a harmonic function with a slightly different frequency (see also Maarse & Thomassen, 1987).

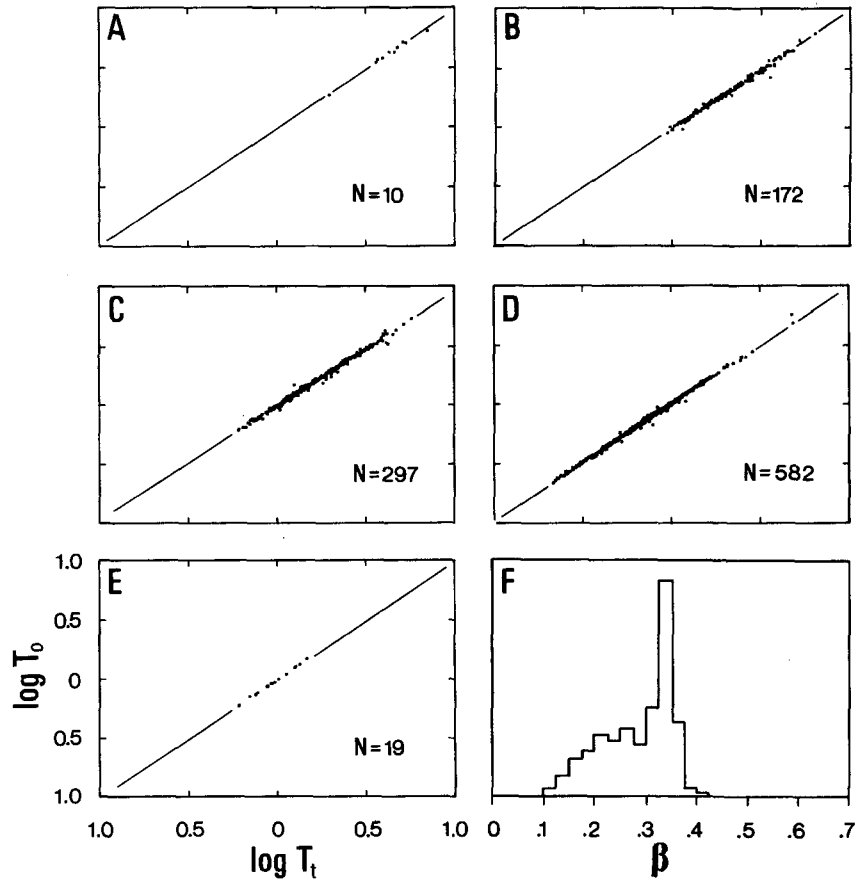


Figure 10. Test of internal consistency. (Panels A-E denote log-log scatterplots of the average period of the movement for each trial [T_e] versus the corresponding theoretical value [T_t] predicted by the GLEM model. The 1,080 data points for all recorded trials were distributed in five groups according to the corresponding value of the exponent β : A, $0 \leq \beta \leq .1$; B, $.1 \leq \beta \leq .2$; C, $.2 \leq \beta \leq .3$; D, $.3 \leq \beta \leq .4$; E, $.4 \leq \beta \leq .5$. N is the number of data points in each plot. Lines through the data points are linear regressions [T_e over T_t]. Panel F denotes the frequency distribution of the exponent β for the entire experimental population. The very high correlation between measured and predicted values of the period supports the assumptions underlying the GLEM model.)

Numerical simulation showed that good approximations to an ellipse may be produced with this composition rule and that each pair of opposite segments gives rise to a distinct linear segment in the $\log V$ - $\log R$ plan. These results raise two questions: First, should the two-thirds power law be abandoned as a principled description of the relationship that mutually constrains form and kinematics? Second, can we interpret the fact that in children the exponent β deviates from the reference value $1/3$ on the basis of the frequency-mismatch hypothesis set forth by Wann et al.?

For the first question we must consider both experimental evidence and theoretical arguments. The experimental data that bear directly on this question are the results from the 6 adults shown in Table 1. Simple inspection of the table ought to dispel any doubt about the adequacy of the power-law formulation. In particular, note that β is absolutely independent of the size of the templates and therefore of the average velocity of the movement that increases by almost a factor of 10 between the smallest and the largest ellipse. Clearly, we are

not ruling out the possibility that biomechanical factors introduce an asymmetry in the execution of the four quadrants of the ellipse. Indeed, systematic distortions are known to occur for certain orientations of the workplane (Soechting & Terzuolo, 1986) and are quite visible in at least one of the traces reported by Wann et al. (1988). A similar flattening of the more proximal region of the trajectory was occasionally observed in our experiments as well. The point is that shape asymmetry alone is not sufficient to violate the power law; the asymmetry must be present in the relationship between trajectory and kinematics. Moreover, the specific hypothesis set forth by Wann and his coworkers (temporal skew of the component velocities) predicts large asymmetries in the corresponding acceleration traces, which we never observed in our traces. (As far as we know, no one else has presented evidence of such asymmetries in the case of ellipses.)

Finally, we comment on the aforementioned points (c) and (d). Work in Viviani and Flash (1990) confirms the observation that the two-thirds power law is qualitatively satisfied by

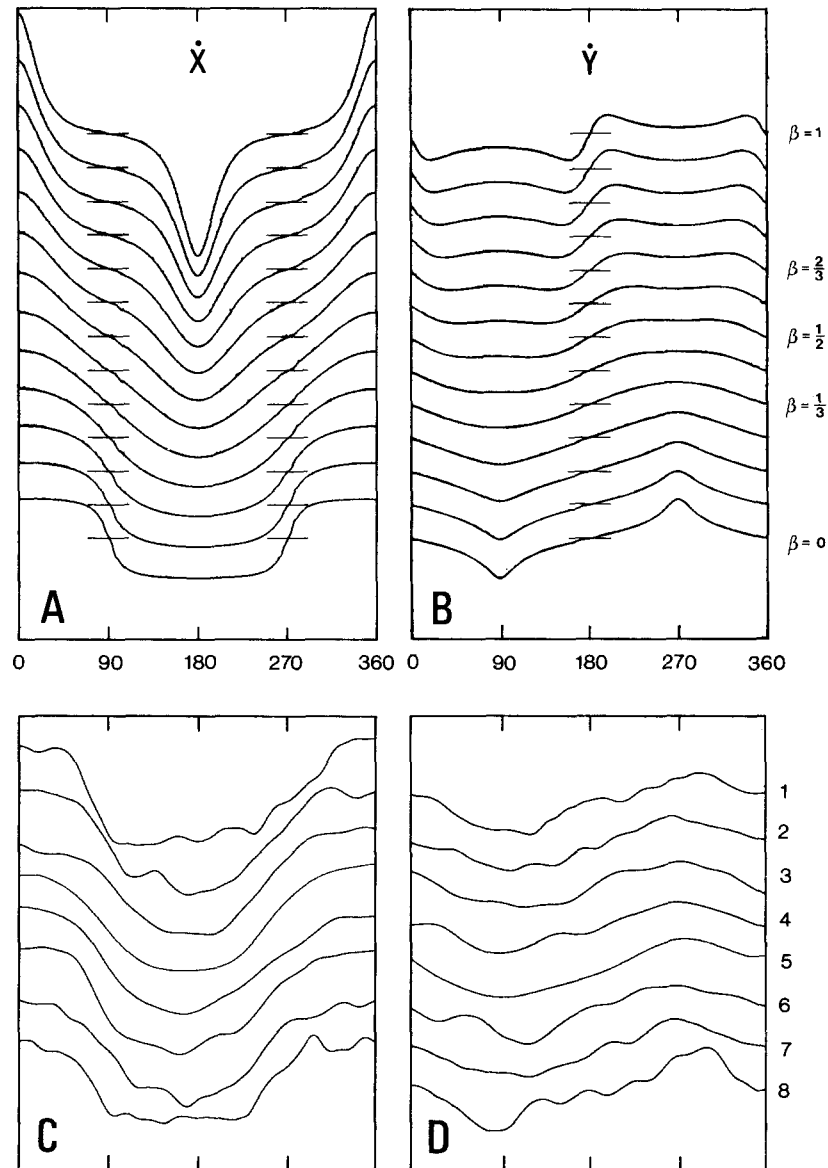


Figure 11. Comparison of velocity profiles with the predictions of the GLEM model. (Horizontal [Panel A] and vertical [Panel B] velocity components of a GLEM were computed by solving Equations 9 and 10 in the Appendix [$K = 10$, $P = 26.51$, $\Sigma = 0.9$, $0 \leq \beta \leq .1$]. The major axis of the trajectory is supposed to be horizontal. The displacement and velocity components are harmonic functions for $\beta = 1/3$ only. Panels C and D denote eight actual velocity-component traces from several subjects that correspond to the following estimates of the exponent β : [1], 0.102; [2], 0.113; [3], 0.124; [4], 0.146; [5], 0.141; [6], 0.159; [7], 0.105; [8], 0.116.)

a class of cost-minimizing kinematic models (Flash & Hogan, 1985; Nelson, 1983). Although we have not verified this directly, a similar agreement most likely exists with a recent dynamic model based on the minimization of torque change (Uno et al., 1989). The observation does not detract from the interest of the power-law formulation, however, for at least two reasons. First, the power law applies to all sorts of trajectories. Only in the case of ellipses (and when the exponent is exactly $1/3$) does it imply the Lissajous mode of gener-

ation. Inflections cannot be handled by the power law in the present form, but work in Viviani and Flash (1990) indicates that a simple generalization of the law exists that covers these exceptional points. Thus, the power law is consistent with but not dependent on the oscillatory models of movement and their possible physiological underpinnings. In addition, the implications of the two formalisms vis à vis the control of movement are prima facie quite distinct. Analytically, the minimum-jerk model solves the following problem: Given

the initial position, the final position, and (optionally) the position of one via point together with the corresponding boundary conditions (time at the via point is not specified), find the trajectory and the law of motion that minimize average quadratic jerk. Thus, the model does not solve the problem of “negotiating a given trajectory while minimizing transients” (Wann et al., 1988, p. 623): The trajectory is part of the solution and depends (sometimes whimsically) on the boundary conditions (Viviani & Flash, 1990). By contrast, the power law admits an interpretation in which the trajectory is truly given a priori and the control system uses the form-kinematics constraint to generate the law of motion along this trajectory (the law of motion is the function $l = l(t)$ that describes the increase with time of the curvilinear coordinate). Because the two formulations are related but not mathematically equivalent, it would actually be interesting to discover what global cost function is minimized by a system that complies with the power law. At any rate, on the basis of our data in adult subjects and the considerations presented earlier, we believe that there are no reasons to abandon the power-law approach for characterizing adult performances.

We now turn to the second question, namely the interpretation of the deviations of the β value in children. A priori, the velocity-skew hypothesis invoked previously appears inadequate because the conditions for the skew to appear (at least in adults) are not met in our experiments. Even the youngest children, who deviated the most from the required eccentricity, never produced ellipses with Σ less than 0.8, whereas in the experiments by Wann et al. (1988) the deviations of the exponent β is only appreciable for $\Sigma = 0.6$. Moreover (as discussed before), neither the velocity (see Figure 11) nor the acceleration traces showed evidence of the skew asymmetry that is supposed to be the cause of the exponent variability. It is still possible, however, to relate the deviations of β to the stiffness of the biochemical system, as Wann et al. suggested. As shown in the Appendix, a GLEM can be described as the motion of a mass-spring system in which the relationship between displacement and force is nonlinear: $d^2x/dt^2 + F(x, P, \Sigma, K_0, \alpha, \beta) = 0$. The incremental stiffness term (dF/dx) of the system is constant only if $\beta = 1/3$. For $\beta < 1/3$ the restoring force is a positively accelerated function of the displacement from the equilibrium point. Consequently, one could speculate that the increase with age of the power-law exponent and the associated changes in the kinematics of the movement are the overt manifestation of a maturational process through which the neuromuscular system evolves toward a linear behavior.

Even if this suggestion has some validity, it is totally unwarranted on the basis of simple behavioral data to advance hypotheses of the means by which the motor-control system achieves such a linearization. We maintain, however, that the analysis of children's performances has permitted us to recast the power-law formulation in a broader perspective by demonstrating that such a formulation is not a mere restatement of the oscillatory theory. A simple relationship between form and kinematics is preserved throughout the evolution of the motor performance with age, and a minimal generalization of the original power law proved sufficient to describe one important aspect of this evolution.

Relationship Between Isochrony and Power Law

In essence, the two-thirds power law posits a factorization of the tangential velocity into two components. On the one hand, the velocity-gain factor K expresses the scaling of the average velocity over one unit of motor action and depends on a global geometric parameter of the unit (its linear extent). On the other hand, the curvature term expresses the instantaneous dependence of the velocity on a local differential parameter of the trajectory. The same interpretation carries over to the generalized version of the power law and allows one to deduce the phenomenon of isochrony from these two factors of the velocity. Specifically, in the case of GLEMs it is possible to work out an exact formula (see the Appendix) which predicts that the period T is a power function of the perimeter P of the ellipse: $T = P^{1-\beta-\alpha}G(\beta, \Sigma)/K_0$, where $G(\beta, \Sigma)$ is a known function of its arguments. Recall that the actual relationship between T and P (Figure 4) was described (empirically) by a power function $T = T_0P^\gamma$. Thus, the accuracy of the prediction can be gauged simply by comparing the exponent γ with $1 - \beta - \alpha$ and comparing the multiplicative constant T_0 with $G(\beta, \Sigma)/K_0$, respectively. To carry out this comparison, we characterize the performance of each subject (children and adults) as follows. We computed the exponents α and γ and the coefficients K_0 and T_0 by linear regressions through the 20 data points (two repetitions and 10 perimeters) that corresponded to all of the trials recorded from that subject. The exponent β was estimated by the average of the slope parameter in the $\log V - \log R$ regression over the same trials. The parameter Σ was estimated as the average eccentricity of all ellipses. Finally, the observed values γ and T_0 were regressed linearly over the corresponding predictors $1 - \alpha - \beta$ and $G(\beta, \Sigma)/K_0$. The results were quite satisfactory (γ : slope = 0.990, intercept = 0.022, correlation = .977; T_0 : slope = 0.970, intercept = -0.040, correlation = .988). The fact that both of the idiosyncratic variations of α , β , γ , Σ , K_0 and T_0 from subject to subject and their systematic changes across age were mutually related as predicted supports the notion that isochrony is rooted in the specific factorization of the tangential velocity expressed by the generalized power law.

Clearly, the aforementioned deduction of isochrony from more primitive concepts is not a true explanation of the phenomenon. One crucial effect—the dependence of the velocity-gain factor on the linear extent of the unit of motor action—is still an empirical fact waiting for a satisfactory psychophysiological interpretation. We maintain, however, that such a deduction is useful insofar as it permits one to state the problem of isochrony in a conceptual framework that is independent of the specific trajectory being traced. It is then possible not only to compare the case of simple figures such as those considered here with that of complex patterns formed by more than one unit (Viviani & Cenzato, 1985) but also to investigate nonperiodic movements. In particular, we are now testing the proposed factorization of the tangential velocity in the case of continuous extemporaneous movements for which the notions of perimeter and period have no meaning (Viviani & Schneider, 1990). Moreover, the separation within each unit of motor action of the role of the linear extent from that of the curvature ought to make it easier to

test the hypothesis that isochronous behavior is an emerging property of the minimum-jerk model (Flash & Hogan, 1985).

Are the Constraints Innate or Learned?

Qualitatively, the results that concern the relationship between curvature and instantaneous velocity confirm the conclusion reached in the discussion of isochrony: This structural constraint is already active at a relatively early stage of development. Moreover, the quantitative differences between the parameters of the $R-V$ relation in children and adults are roughly of the same order of magnitude as those observed in the establishment of isochrony. Thus the genesis of these two properties of the motor-control system calls for similar comments. Constraints may be progressively imposed, through learning, on an unconstrained initial state. For learning to occur, however, it must be driven either by prescription and imitation or by some cost-effectiveness trade-off. The first hypothesis runs into the well-known difficulty that has already been pointed out in other domains: There is not enough environmental evidence to provide the developing baby with a consistent body of norms. Isochrony and the two-thirds power law manifest themselves as accurate covariations of geometric and kinematic parameters that are very difficult to apprehend perceptually. In fact, recent visual experiments (Viviani & Stucchi, 1989, 1990) demonstrated that both the geometry and the velocity of movements that do not comply with the two-thirds power law are perceptually distorted. If anything, these results suggest that visual perception is influenced by some specific properties of the motor system and not vice versa. On the other hand, proprioceptive reafferences might be sufficiently accurate to provide a veridical representation of the constraints when they are present, but clearly they cannot alone provide the appropriate input for a learning process. The second hypothesis is supported by certain cost-minimizing models that predict both a correlation between curvature and velocity and a certain degree of isochrony. Note, however, that the cost being minimized—be it jerk or torque change (see the previous discussion)—is hardly the kind of feedback information considered effective for learning. Finally, the learning hypothesis seems to be inconsistent with the fact that certain performance parameters undergo regression phases in their evolution with age (cf. Figures 2, 5, and 7). Indeed, whenever regression phases have been documented in motor, perceptual, and cognitive performances (Bever, 1982; Lockman & Ashmead, 1982; Mounoud et al., 1985; Werker & Tees, 1983) it proved impossible to ascribe them to a learning process. This difficulty is all the more serious in our case, that temporal, geometric, and relational parameters exhibit regressions at different ages.

As an alternative to the learning hypothesis, consider the possibility that the $P-V$ and $R-V$ covariations emerge epigenetically as part of the general process of motor development that takes place in the earliest years of life. The question was raised by Sciaky et al. (1987) in a developmental study of the relationship between curvature and velocity. These authors assumed the validity of the two-thirds power law at all ages and concentrated on the coefficient of linear correlation between $\log V$ and $\log R^{1/3}$. The evolution between 5 and 12

years of the correlation for freehand elliptic movements was compatible with an unconstrained initial state (Sciaky et al., 1987, p. 524, Figure 4), as the epigenetic hypothesis suggests. Nevertheless, the results of this study are subject to caution because as we have shown the assumption that $\beta = 1/3$ at all ages is incorrect. This may explain why the correlations measured by Sciaky et al. are much lower than those reported here (upper panel, Figure 7; see also Wann, 1989). Although only appropriate measurements in babies and younger children may be able to settle the point, we maintain that the age-related trends in our experiments make the epigenetic hypothesis somewhat unlikely. In fact, a truly unconstrained initial state implies a zero correlation both between average velocity and perimeter and between instantaneous velocity and radius of curvature. This is certainly not suggested by backward extrapolation of the correlation plots in Figures 5 and 7: For these plots to converge toward 0, we need to postulate a fairly abrupt evolution of the correlation within the first years of life, which, although possible, is unusual in the context of motor development (Mounoud, 1986).

On the basis of our evidence, it seems reasonable to accept the provisional conclusion that the psychophysiological bases for the establishment of the constraints are part of the genetic endowment. The fact that the measurable manifestations of these constraints evolve over the entire range of age covered by the experiments and that the evolution is not yet completed at the inception of puberty demonstrates, however, that the full deployment of such an inborn characteristic of the motor-control system is a surprisingly elaborate process.

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Appendix

The GLEM Model

We derive the predictions of the GLEM model that are relevant to the discussion of the experimental data. Assume that at all points of the trajectory that are sufficiently removed from inflections, the power law

$$V(t) = K R(t)^\beta, \quad \beta \geq 0, \quad (1)$$

provides a good approximation of the actual relationship between tangential velocity V and radius of curvature R . From kinematics we know that

$$V(t) = [(dx/dt)^2 + (dy/dt)^2]^{1/2} \quad (2)$$

and

$$R(t) = V^3 / [(d^2y/dt^2)(dx/dt) - (d^2x/dt^2)(dy/dt)]. \quad (3)$$

By substituting Equations 2 and 3 into 1, expressing the time derivatives of the y components in terms of the time derivatives of the x component, and rearranging, we get

$$(dx/dt)(d^2y/dx^2)^\beta [1 + (dy/dx)^2]^{(1-\beta)/2} = K. \quad (4)$$

For a given trajectory $y = f(x)$, the derivatives dy/dx and d^2y/dx^2 are explicit functions of x , and Equation 4 becomes a nonlinear differential equation of the first order in t :

$$dx/dt = W(x, t, \beta, K). \quad (5)$$

In the case of GLEMs, the trajectory is an ellipse with semiaxes A

and B ($A \geq B$):

$$y = \pm(B/A)(A^2 - x^2)^{1/2}, \quad (6)$$

$$dy/dx = \pm(Bx/A)(A^2 - x^2)^{-1/2}, \quad (7)$$

and

$$d^2y/dx^2 = \pm AB(A^2 - x^2)^{-3/2}. \quad (8)$$

By substituting Equations 7 and 8 into Equation 4 and expressing the result in the form of Equation 5, one finally gets

$$dx/dt = K(AB)^{-\beta} (A^2 - x^2)^{1/2} / (A^2 - \Sigma^2 x^2)^{(1-\beta)/2}, \quad -A \leq x \leq A, \quad \Sigma = (1 - (B/A)^2)^{1/2}, \quad \beta \geq 0. \quad (9)$$

Similar arguments lead to an analogous expression for the y component:

$$dy/dt = K(AB)^{-\beta} (B^2 - y^2)^{1/2} / (B^2 - \Sigma^{*2} y^2)^{(1-\beta)/2}, \quad -B \leq y \leq B, \quad \Sigma^{*2} = (1 - (A/B)^2)^2. \quad (10)$$

The values dx/dt and dy/dt vanish whenever $x = \pm A$ and $y = \pm B$, respectively; and they have an extremum whenever $x = 0$ and $y = 0$, respectively. Thus, Equations 9 and 10 have periodic solutions of equal frequency for all values of the parameters A , B , β , and K . These solutions can either be expressed as series of Jacobi elliptic functions or be computed by numeric integration. Figure A1 shows five illustrative examples of the solutions $x(t)$ and $y(t)$ that correspond to $\Sigma =$

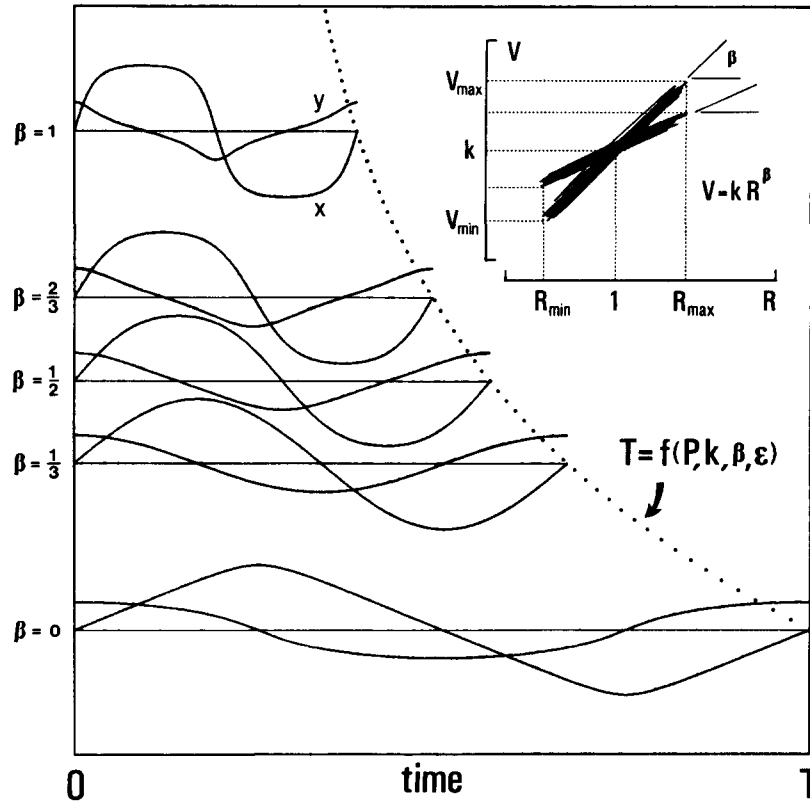


Figure A1. Simulated examples of GLEMs: horizontal (x) and vertical (y) components of an elliptic movement that satisfies the generalized power law (Equation 1). (For each of the five indicated β values the components were computed by solving the differential Equations 9 and 10 in the Appendix. Zero lines for plotting the components are spaced according to the corresponding values of β . In this simulation the parameters of the trajectory of the movement were set as follows: $P = 26.51$ cm, $\Sigma = 0.9$, and the major axis was horizontal. The velocity-gain factor K was set to 10. Notice that the components are harmonic functions only when $\beta = 1/3$. The dotted line represents the function $T = f[P, K, \beta, \Sigma]$ [Equation 13]. The inset diagram illustrates the significance of the parameter β .)

0.9, $P = 26.51$, $K = 10$, and the five indicated values of β . When $\beta = 1/3$ (and only then) both Cartesian components of a GLEM are harmonic functions. Even relatively small deviations from this special value result in noticeable departures from sine and cosine functions, which become even more apparent if one considers the velocity components and scales the periods as in Figure 11. As for the period of the solutions, we notice that Equations 9 and 10 are separable. With the change of variable $x = A \sin \Phi$ and through the use of the notation

$$\Gamma(\beta, \Sigma) = \int_0^{\pi/2} [(1 - \Sigma^2 \sin^2 \Phi)^{(1 - 3\beta)/2}] d\Phi, \quad (11)$$

either one of the two differential equations yields easily:

$$T = 4A^{1 - 2\beta} B^\beta \Gamma(\beta, \Sigma) / K, \quad (12)$$

which generalizes the result obtained in Viviani and Cenzato (1985). Through the use of the complete elliptic integral of the second kind, $E(\pi/2, \Sigma)$, the semiaxes of the ellipse can be expressed as a function of the perimeter P and of the eccentricity. Thus, the period can also be written as

$$T = 4(P^{1 - \beta} / K)(1 - \Sigma^2)^{\beta/2} [4E(\pi/2, \Sigma)]^{\beta - 1} \Gamma(\beta, \Sigma) = f(P, K, \beta, \Sigma). \quad (13)$$

For a given ellipse, the period of a GLEM depends jointly on the velocity-gain factor K and on the exponent of the relationship between R and V . For a fixed value of K , the period T can be either an increasing or a decreasing function of β , depending on the range of oscillation of the radius of curvature (see inset in Figure A1). It can be shown that for $\Sigma = 0.9$, $dT/d\beta < 0$ whenever the harmonic mean of the minimum and maximum radius of curvature is greater than 0.172. This condition was satisfied by all of our templates. The dotted line in Figure A1 represents the function $T = f(P, K, \beta, \Sigma)$ ($0 \leq \beta \leq 1$). Equation 13 provides the basis both for the test of internal consistency described in the text and for predicting the relationship between the perimeter of the ellipse and the period of the movement (Figure 5). From the data in Figure 10 we have derived the empirical expression $K = K_0 P^\alpha$, which relates the velocity-gain factor to the perimeter. By substituting this expression into Equation 13 one obtains

$$T = [G(\beta, \Sigma) / K_0] P^{1 - \beta - \alpha}. \quad (14)$$

Finally, we show that GLEMs can be interpreted as the motion of an undamped nonlinear mass-spring system. Differentiating both sides of Equation 9 with respect to time and eliminating first derivatives by again substituting Equation 9 in the result gives

$$d^2x/dt^2 + F(x, P, \Sigma, K_0, \alpha, \beta) = 0, \quad (15)$$

where F is the nonlinear restoring force. The "stiffness" of the system is then obtained by differentiating F with respect to x :

dF/dx

$$\begin{aligned} &= K^2 \{ (A^2 - \Sigma^2 x^2)^{3\beta - 1} \\ &+ (3\beta - 1) \Sigma^2 (A^2 - 5x^2) (A^2 - \Sigma^2 x^2)^{3\beta - 2} \\ &- 2(3\beta - 1)(3\beta - 2) \Sigma^4 (A^2 - x^2) \\ &\times (A^2 - \Sigma^2 x^2)^{3\beta - 3} \} / (AB)^{2\beta}. \end{aligned} \quad (16)$$

Easy calculations show that dF/dx is constant for $\beta = 1/3$, increasing for $0 \leq \beta \leq 1/3$, and decreasing for $\beta \geq 1/3$. Expressions similar to Equations 15 and 16 can be derived for the y component.

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Harvey Appointed Editor of *Contemporary Psychology*, 1992–1997

The Publications and Communications Board of the American Psychological Association announces the appointment of John H. Harvey, University of Iowa, as editor of *Contemporary Psychology* for a 6-year term beginning in 1992.

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