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A Numerical Method for Solving the Hyperbolic Telegraph Equation

Mehdi Dehghan, Ali Shokri
Department of Applied Mathematics, Faculty of Mathematics and Computer Science, Amirkabir University of Technology, Tehran, Iran

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Recently, it is found that telegraph equation is more suitable than ordinary diffusion equation in modelling reaction diffusion for such branches of sciences. In this article, we propose a numerical scheme to solve the one-dimensional hyperbolic telegraph equation using collocation points and approximating the solution using thin plate splines radial basis function. The scheme works in a similar fashion as finite difference methods. The results of numerical experiments are presented, and are compared with analytical solutions to confirm the good accuracy of the presented scheme. Numer Methods Partial Differential Eq 24: 1080–1093, 2008

Keywords: collocation; radial basis function (RBF); second-order hyperbolic telegraph equation; thin plate splines (TPS)

I. INTRODUCTION

In the present work we are dealing with the numerical approximation of the following second order hyperbolic problem:

$$\frac{\partial^2 u}{\partial t^2} + \alpha \frac{\partial u}{\partial t} + \beta u = \frac{\partial^2 u}{\partial x^2} + f(x,t), \tag{1.1}$$

where α and β are known constant coefficients. Equation (1.1), referred to as second-order telegraph equation with constant coefficients, models mixture between diffusion and wave propagation by introducing a term that accounts for effects of finite velocity to standard heat or mass transport equation [1]. However, Eq. (1.1) is commonly used in signal analysis for transmission and propagation of electrical signals [2] and also has applications in other fields (see [3] and the references therein).

In recent years, much attention has been given in the literature to the development, analysis, and implementation of stable methods for the numerical solution of second-order hyperbolic

Correspondence to: Mehdi Dehghan, Department of Applied Mathematics, Faculty of Mathematics and Computer Science, Amirkabir University of Technology, No. 424 Hafez Ave., Tehran, Iran (e-mail: mdehghan@aut.ac.ir)

equations, (see [4] and the references therein). Recently, Mohanty et al. [5, 6] developed new three-level implicit unconditionally stable alternating direction implicit schemes for the two and three-space-dimensional linear hyperbolic equations. These schemes are second-order accurate both in space and time.

Of concern are suspension flows. These combine directed and random motion and are traditionally modeled by parabolic partial differential equations. Sometimes they can be better modeled (in terms of fitting the data generated by certain blood flow experiments) by hyperbolic equations such as the telegraph equation, which has parabolic (or analytic) asymptotic [7]. In particular, the experimental results described in [8, 9] seem to be better modeled by the telegraph equation than by the heat equation. Also the interested reader can see [10] for an application of the model.

The existence of time-bounded solutions of nonlinear bounded perturbations of the telegraph equation with Neumann boundary conditions has recently been considered in [11]. The approach is based upon a Galerkin method combined with the use of some Lyapunov functionals.

Finite difference methods are known as the first techniques for solving partial differential equations [12, 13]. Even though these methods are very effective for solving various kinds of partial differential equations, conditional stability of explicit finite difference procedures and the need to use large amount of CPU time in implicit finite difference schemes limit the applicability of these methods. Furthermore, these methods provide the solution of the problem on mesh points only [14] and accuracy of these well known techniques is reduced in non-smooth and nonregular domains.

To avoid the mesh generation, in recent years meshless techniques have attracted attention of researchers. In a meshless (mesh free) method a set of scattered nodes are used instead of meshing the domain of the problem. Some meshless schemes are the element free Galerkin method, the reproducing kernel particle, the local point interpolation and etc. For more descriptions see [15] and references therein.

In the last 20 years, the radial basis functions method is known as a powerful tool for scattered data interpolation problem. The use of radial basis functions as a meshless procedure for numerical solution of partial differential equations is based on the collocation scheme. Because of the collocation technique, this method does not need to evaluate any integral. The main advantage of numerical procedures which use radial basis functions over traditional techniques is meshless property of these methods. Radial basis functions (RBFs) are used actively for solving partial differential equations. For example see [16, 17].

In the last decade, the development of the RBFs as a truly meshless method for approximating the solutions of PDEs has drawn the attention of many researchers in science and engineering. One of the domain-type meshless methods, the so-called Kansa's method developed by Kansa in 1990 [16, 18], is obtained by directly collocating the RBFs, particularly the multiquadric (MQ), for the numerical approximation of the solution.

Kansa's method was recently extended to solve various ordinary and partial differential equations including 1D nonlinear Burgers' equation [19] with shock wave, shallow water equations for tide and currents simulation [20], heat transfer problems [17], KdV equation [21], and free boundary problems [22, 23]. Fasshauer [24] later modified Kansa's method to a Hermite type collocation method for the solvability of the resultant collocation matrix.

The traditional RBFs are globally defined functions which result in a full resultant coefficient matrix. This hinders the application of the RBFs to solve large scale problems due to severe ill-conditioning of the coefficient matrix. To tackle this ill-conditioning problem, a new class of compactly supported RBFs were constructed by [25]. For the theoretical developments of the RBFs in scattered data interpolation, Madych and Nelson [26, 27] showed that the RBF-MQ interpolant employs exponential convergence with minimal semi-norm errors. Recently, Franke and Schaback [28,29] provided a theoretical proof in using the RBFs for the numerical solutions of PDEs. More recently, Hon and Wu [30] gave a theoretical justification in combining the RBFs with those advanced techniques of domain decomposition, multilevel/multigrid, Schwarz iterative schemes, and preconditioning in the FEM discipline.

This article presents a new numerical scheme to solve the second-order hyperbolic telegraph equation [31,32] using the collocation method and approximating directly the solution using thin plate splines radial basis function (Kansa's method). The scheme is similar to finite-difference methods [33–35].

The layout of the article is as follows: In Section 2 we show that how we use the radial basis functions to approximate the solution. In Section 3 we apply the method on the hyperbolic telegraph equation. The results of numerical experiments are presented in Section 4. Section 5 is dedicated to a brief conclusion. Finally some references are introduced at the end. Note that we have computed the numerical results by Matlab programming.

II. RADIAL BASIS FUNCTION APPROXIMATION

The approximation of a distribution $u(\mathbf{x})$, using radial basis functions, may be written as a linear combination of N radial functions; usually it takes the following form:

$$u(\mathbf{x}) \simeq \sum_{j=1}^{N} \lambda_j \varphi(\mathbf{x}, \mathbf{x}_j) + \psi(\mathbf{x}) \quad \text{for } \mathbf{x} \in \Omega \subset \mathbb{R}^d, \tag{2.1}$$

where N is the number of data points, $\mathbf{x} = (x_1, x_2, \ldots, x_d)$, d is the dimension of the problem, λ's are coefficients to be determined and φ is the radial basis function. Equation (2.1) can be written without the additional polynomial ψ. In that case φ must be *unconditionally positive definite* to guarantee the solvability of the resulting system (e.g., Gaussian or inverse multiquadrics). However, ψ is usually required when φ is *conditionally positive definite*, i.e., when φ has a polynomial growth towards infinity. For two examples we can mention thin plate splines and multiquadrics. We will use thin plate splines for the numerical scheme introduced in Section 3. The generalized thin plate splines (TPS) defined as:

$$\varphi(\mathbf{x}, \mathbf{x}_j) = \varphi(r_j) = r_j^{2m} \log(r_j), \quad m = 1, 2, 3, \ldots, \tag{2.2}$$

where $r_j = \|\mathbf{x} - \mathbf{x}_j\|$ is the Euclidean norm.

Since φ in (2.2) is C^{2m-1} continuous, so higher order thin plate splines must be used for higher order partial differential operators. For the one-dimensional hyperbolic telegraph equation, $m = 2$ is used for thin plate splines (i.e., second order thin plate splines).

If $\mathcal{P}_q^d$ denotes the space of d-variate polynomials of order not exceeding q, and letting the polynomials $P_1, \ldots, P_m$ be the basis of $\mathcal{P}_q^d$ in $\mathbb{R}^d$, then the polynomial $\psi(\mathbf{x})$, in Eq. (2.1), is usually written in the following form:

$$\psi(\mathbf{x}) = \sum_{i=1}^{m} \zeta_i P_i(\mathbf{x}), \tag{2.3}$$

where $m = (q - 1 + d)!/(d!(q - 1)!)$.

To determine the coefficients $(\lambda_1, \dots, \lambda_N)$ and $(\zeta_1, \dots, \zeta_m)$, the collocation method is used. However, in addition to the N equations resulting from collocating (2.1) at the N points, extra m equations are required. This is insured by the m conditions for (2.1),

$$\sum_{j=1}^{N} \lambda_j P_i(\mathbf{x}_j) = 0, \quad i = 1, \dots, m. \tag{2.4}$$

In a similar representation as (2.1), for any linear partial differential operator $\mathcal{L}$, $\mathcal{L}u$ can be approximated by

$$\mathcal{L}u(\mathbf{x}) \simeq \sum_{j=1}^{N} \lambda_j \mathcal{L}\varphi(\mathbf{x}, \mathbf{x}_j) + \mathcal{L}\psi(\mathbf{x}). \tag{2.5}$$

III. THE HYPERBOLIC TELEGRAPH EQUATION

Let us consider the following one-dimensional hyperbolic telegraph equation:

$$\frac{\partial^2 u}{\partial t^2} + \alpha \frac{\partial u}{\partial t} + \beta u = \frac{\partial^2 u}{\partial x^2} + f(x,t), \quad x \in \Omega = [a,b] \subset \mathbb{R}, \ \ 0 < t \leq T, \tag{3.1}$$

with the initial conditions

$$\begin{cases} u(x,0) = g_1(x), & x \in \Omega \\ u_t(x,0) = g_2(x), & x \in \Omega \end{cases}, \tag{3.2}$$

and Dirichlet boundary condition

$$u(x,t) = h(x,t), \quad x \in \partial\Omega, \ \ 0 < t \leq T, \tag{3.3}$$

where α and β are known constant coefficients, f, g_1, g_2, and h are known functions, and the function u is unknown.

First, let us discretize (3.1) according to the following θ-weighted scheme

$$\frac{u(x,t+\delta t) - 2u(x,t) + u(x,t-\delta t)}{(\delta t)^2} + \alpha \frac{u(x,t+\delta t) - u(x,t-\delta t)}{2\delta t}$$
$$= \theta[\nabla^2 u(x,t+\delta t) - \beta u(x,t+\delta t)] + (1-\theta)[\nabla^2 u(x,t) - \beta u(x,t)] + f(x,t+\delta t), \tag{3.4}$$

where ∇ is the gradient differential operator, $0 \leq \theta \leq 1$, and δt is the time step size. Rearranging (3.4), using the notation $u^n = u(x,t^n)$ where $t^n = t^{n-1} + \delta t$, we obtain

$$\left(1 + \frac{\alpha \delta t}{2} + \beta\theta(\delta t)^2\right) u^{n+1} - \theta(\delta t)^2 \nabla^2 u^{n+1}$$
$$= (2 - \beta(1-\theta)(\delta t)^2) u^n + (1-\theta)(\delta t)^2 \nabla^2 u^n + \left(\frac{\alpha \delta t}{2} - 1\right) u^{n-1} + (\delta t)^2 f^{n+1}. \tag{3.5}$$

Assuming that there are a total of $(N-2)$ interpolation points, $u(x,t^n)$ can be approximated by

$$u^n(x) \simeq \sum_{j=1}^{N-2} \lambda_j^n \varphi(r_j) + \lambda_{N-1}^n x + \lambda_N^n. \tag{3.6}$$

To determine the interpolation coefficients $(\lambda_1, \lambda_2, \dots, \lambda_{N-1}, \lambda_N)$, the collocation method is used by applying (3.6) at every point $x_i, i = 1, 2, \dots, N-2$. Thus we have

$$u^n(x_i) \simeq \sum_{j=1}^{N-2} \lambda_j^n \varphi(r_{ij}) + \lambda_{N-1}^n x_i + \lambda_N^n, \tag{3.7}$$

where $r_{ij} = \sqrt{(x_i - x_j)^2}$. The additional conditions due to (2.4) are written as

$$\sum_{j=1}^{N-2} \lambda_j^n = \sum_{j=1}^{N-2} \lambda_j^n x_j = 0. \tag{3.8}$$

Writing (3.7) together with (3.8) in a matrix form we have

$$[u]^n = \mathbf{A}[\lambda]^n, \tag{3.9}$$

where $[u]^n = [u_1^n\, u_2^n \cdots u_{N-2}^n\, 0\, 0]^T$, $[\lambda]^n = [\lambda_1^n\, \lambda_2^n \cdots \lambda_N^n]^T$ and $\mathbf{A} = [a_{ij}, 1 \le i, j \le N]$ is given by

$$\mathbf{A} = \begin{bmatrix} \varphi_{11} & \cdots & \varphi_{1(N-2)} & x_1 & 1 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \varphi_{(N-2)1} & \cdots & \varphi_{(N-2)(N-2)} & x_{N-2} & 1 \\ x_1 & \cdots & x_{N-2} & 0 & 0 \\ 1 & \cdots & 1 & 0 & 0 \end{bmatrix}. \tag{3.10}$$

Assuming that there are $p < (N-2)$ internal points and $(N-2-p)$ boundary points, then the $(N \times N)$ matrix $\mathbf{A}$ can be splitted into: $\mathbf{A} = \mathbf{A}_d + \mathbf{A}_b + \mathbf{A}_e$, where

$$\begin{aligned} \mathbf{A}_d &= [a_{ij} \text{ for } (1 \le i \le p, 1 \le j \le N) \text{ and 0 elsewhere}], \\ \mathbf{A}_b &= [a_{ij} \text{ for } (p+1 \le i \le N-2, 1 \le j \le N) \text{ and 0 elsewhere}], \\ \mathbf{A}_e &= [a_{ij} \text{ for } (N-1 \le i \le N, 1 \le j \le N) \text{ and 0 elsewhere}]. \end{aligned} \tag{3.11}$$

Using the notation $\mathcal{L}\mathbf{A}$ to designate the matrix of the same dimension as $\mathbf{A}$ and containing the elements $\tilde{a}_{ij}$, where $\tilde{a}_{ij} = \mathcal{L}a_{ij}, 1 \le i, j \le N$, then Eq. (3.5) together with (3.3) can be written, in the matrix form, as

$$B[\lambda]^{n+1} = C[\lambda]^n + \left(\frac{\alpha\delta t}{2} - 1\right)[u_d]^{n-1} + (\delta t)^2[f]^{n+1} + [H]^{n+1}, \tag{3.12}$$

where

$$B = \left(1 + \frac{\alpha\delta t}{2} + \beta\theta(\delta t)^2\right) A_d - \theta(\delta t)^2\nabla^2 A_d + A_b + A_e,$$

$$C = (2 - \beta(1-\theta)(\delta t)^2)A_d + (1-\theta)(\delta t)^2\nabla^2 A_d,$$

$$[u_d]^{n-1} = \left[u_1^{n-1} \cdots u_p^{n-1} 0 \cdots 0\right]^T, [f]^{n+1} = \left[f_1^{n+1} \cdots f_p^{n+1} 0 \cdots 0\right]^T,$$

and $[H]^{n+1} = [0 \cdots 0\, h_{p+1}^{n+1} \cdots h_{N-2}^{n+1}\, 0\, 0]^T$. Equation (3.12) is obtained by combining (3.5), which applies to the domain points, while (3.3) applies to the boundary points.

At $n = 0$ the Eq. (3.12) has the following form:

$$B[\lambda]^1 = C[\lambda]^0 + \left(\frac{\alpha\delta t}{2} - 1\right)[u_d]^{-1} + (\delta t)^2[f]^1 + [H]^1. \tag{3.13}$$

To approximate u^{-1} the second initial condition can be used. For this purpose we discretize the second initial condition as

$$\frac{u^1(x) - u^{-1}(x)}{2\delta t} = g_2(x), \quad x \in \Omega. \tag{3.14}$$

Writing (3.13) together with (3.14) we have

$$\left(B + \left(1 - \frac{\alpha\delta t}{2}\right)A_d\right)[\lambda]^1 = C[\lambda]^0 + (2 - \alpha\delta t)\delta t[G] + (\delta t)^2[f]^1 + [H]^1, \tag{3.15}$$

where $[G] = [(g_2)_1 \cdots (g_2)_p\, 0 \cdots 0]^T$.

Since the coefficient matrix is unchanged in time steps, we use the LU factorization to the coefficient matrix only once and use this factorization in our algorithm.

Remark. Although Eq. (3.12) is valid for any value of $\theta \in [0, 1]$, we will use $\theta = 1/2$ (the famous Crank-Nicolson scheme).

IV. NUMERICAL RESULTS

In this section we present some numerical results to test the efficiency of the new scheme for solving the hyperbolic telegraph equation.

A. Example 1

In this example, we consider the hyperbolic telegraph Eq. (3.1) with $\alpha = 1$ and $\beta = 1$ in the interval $0 \le x \le 4$. The initial conditions are given by

$$\begin{cases} u(x,0) = g_1(x) = e^x, & 0 \le x \le 4 \\ u_t(x,0) = g_2(x) = -e^x, & 0 \le x \le 4 \end{cases}, \tag{4.1}$$

and the analytical solution is given in [32] as

$$u(x,t) = \exp(x - t). \tag{4.2}$$

In this case $f(x,t) = 0$. We extract the boundary function $h(x,t)$ from the exact solution. The L_∞ and L_2 errors and Root-Mean-Square (RMS) of errors are obtained in Table I for $t = 1, 2, 3, 4$, and 5. The graph of analytical and estimated functions for $t = 5$ is given in Fig. 1. We also draw the space-time graph of the estimated solution in Fig. 2.

TABLE I. Computational domain is [0, 4].

t	L_∞-error	L_2-error	RMS	Time (s)
1	2.2931×10^{-5}	1.7163×10^{-4}	8.5711×10^{-6}	7
2	1.3305×10^{-5}	1.0283×10^{-4}	5.1353×10^{-6}	14
3	6.9011×10^{-6}	4.3509×10^{-5}	2.1727×10^{-6}	21
4	3.2736×10^{-6}	1.8865×10^{-5}	9.4211×10^{-7}	28
5	3.0766×10^{-6}	2.0235×10^{-5}	1.0105×10^{-6}	35

L_∞, L_2, and RMS errors, with $dt = 0.001$, $dx = 0.01$.

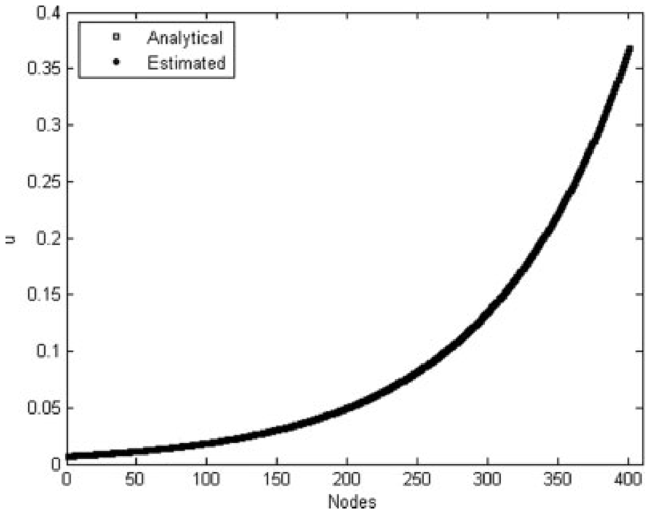

FIG. 1. Analytical and estimated function in $t = 5s$, with $dt = 0.001$ and $dx = 0.01$, for Example 1.

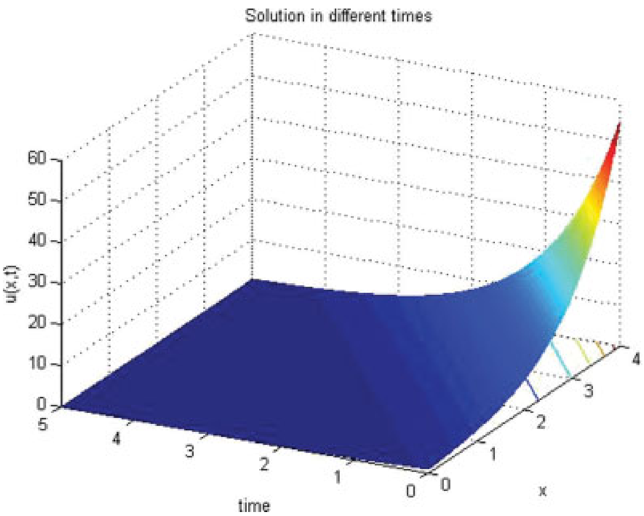

FIG. 2. Space-time graph of the solution up to $t = 5s$, with $dt = 0.001$ and $dx = 0.01$, for Example 1. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

TABLE II. Case 1: $\alpha = 4$, $\beta = 2$, and computational domain is $[0, \pi]$.

t	L_∞-error	L_2-error	RMS	Time(s)
0.5	8.3721×10^{-6}	7.9491×10^{-5}	6.3239×10^{-6}	5
1.0	1.5680×10^{-5}	1.4554×10^{-4}	1.1579×10^{-5}	12
1.5	1.7412×10^{-5}	1.5895×10^{-4}	1.2645×10^{-5}	19
2.0	1.5813×10^{-5}	1.4185×10^{-4}	1.1285×10^{-5}	28

L_∞, L_2, and RMS errors, with $dt = 0.0001$, $dx = 0.02$.

B. Example 2

Consider the hyperbolic telegraph Eq. (3.1) in the interval $0 \le x \le \pi$. The initial conditions are given by

$$\begin{cases} u(x,0) = g_1(x) = \sin(x), & 0 \le x \le \pi \\ u_t(x,0) = g_2(x) = -\sin(x), & 0 \le x \le \pi \end{cases}. \tag{4.3}$$

The analytical solution is given in [4] as

$$u(x,t) = \exp(-t)\sin(x). \tag{4.4}$$

For this example, we solved the equation in two cases $\alpha = 4$, $\beta = 2$ and $\alpha = 6$, $\beta = 2$. For each of these two cases we have

$$f(x,t) = (2 - \alpha + \beta)\exp(-t)\sin(x).$$

We extract the boundary function $h(x,t)$ from the exact solution. The L_∞ and L_2 errors and RMS of errors for these two cases are obtained in Tables II and III for $t = 0.5, 1, 1.5$, and 2. The graph of analytical and estimated functions for the second case at $t = 2$ is given in Fig. 3. We also draw the space-time graph of estimated solution for the second case in Fig. 4.

C. Example 3

In this example, we consider the hyperbolic telegraph Eq. (3.1) with $\alpha = 1$ and $\beta = 1$ in the interval $0 \le x \le 1$. The initial conditions are given by

$$\begin{cases} u(x,0) = g_1(x) = 0, & 0 \le x \le 1 \\ u_t(x,0) = g_2(x) = 0, & 0 \le x \le 1 \end{cases}, \tag{4.5}$$

and the exact solution by [1] is

$$u(x,t) = (x - x^2)t^2\exp(-t). \tag{4.6}$$

TABLE III. Case 2: $\alpha = 6$, $\beta = 2$, and computational domain is $[0, \pi]$.

t	L_∞-error	L_2-error	RMS	Time(s)
0.5	1.5502×10^{-5}	1.4091×10^{-4}	1.1210×10^{-5}	5
1.0	2.7091×10^{-5}	2.4564×10^{-4}	1.9542×10^{-5}	12
1.5	3.0300×10^{-5}	2.7426×10^{-4}	2.1819×10^{-5}	19
2.0	2.9171×10^{-5}	2.6172×10^{-4}	2.0821×10^{-5}	28

L_∞, L_2, and RMS errors, with $dt = 0.0001$, $dx = 0.02$.

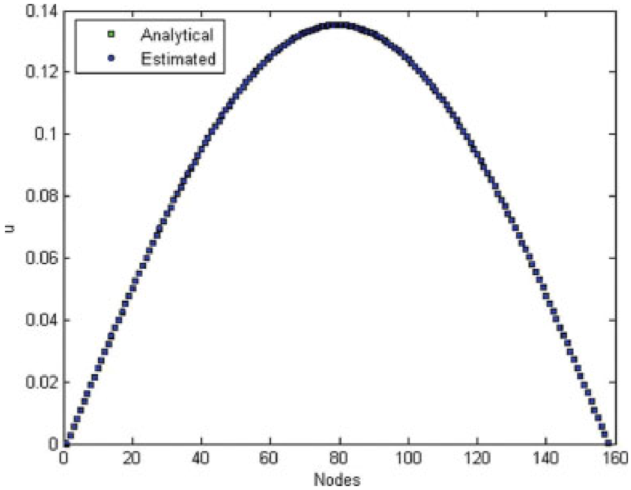

FIG. 3. Analytical and estimated function in $t = 2s$, with $dt = 0.0001$ and $dx = 0.02$, for Example 2 (case 2). [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

The right hand side function is

$$f(x,t) = (2 - 2t + t^2)(x - x^2)\exp(-t) + 2t^2\exp(-t). \tag{4.7}$$

The L_∞ and L_2 errors and RMS of errors are obtained in Table IV for $t = 1, 2, 3, 4$, and 5. The graph of analytical and estimated functions for $t = 5$ and space-time graph of estimated solution are given in Figs. 5 and 6.

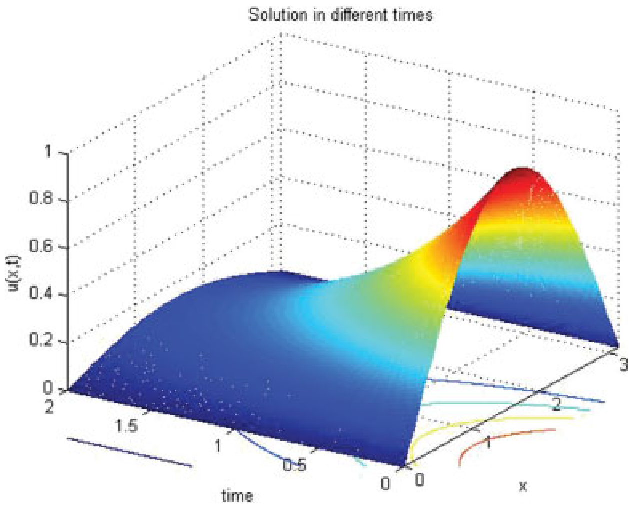

FIG. 4. Space-time graph of the solution up to $t = 2s$, with $dt = 0.0001$ and $dx = 0.02$, for Example 2 (case 2). [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

TABLE IV. Computational domain is [0, 1].

t	L_∞-error	L_2-error	RMS	Time(s)
1	1.8479×10^{-5}	1.4386×10^{-4}	1.4315×10^{-5}	0
2	1.0713×10^{-5}	8.0879×10^{-5}	8.0478×10^{-6}	0
3	1.8161×10^{-5}	1.2944×10^{-4}	1.2880×10^{-5}	1
4	1.6489×10^{-5}	1.1845×10^{-4}	1.1786×10^{-5}	1
5	1.0455×10^{-5}	7.5545×10^{-5}	7.5170×10^{-6}	2

L_∞, L_2, and RMS errors, with $dt = 0.001$, $dx = 0.01$.

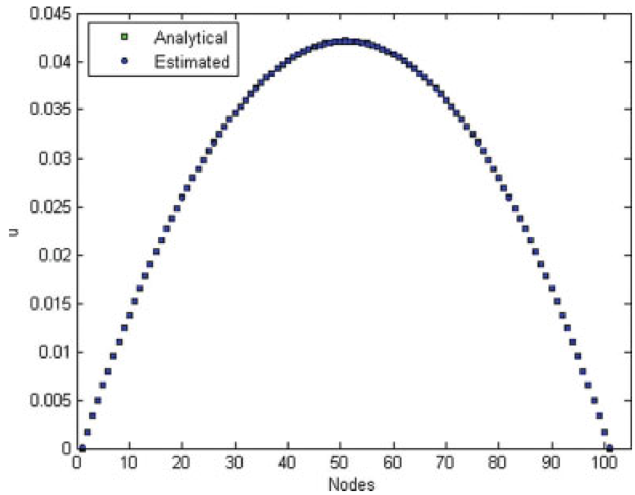

FIG. 5. Analytical and estimated function in $t = 5s$, with $dt = 0.001$ and $dx = 0.01$, for Example 3. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

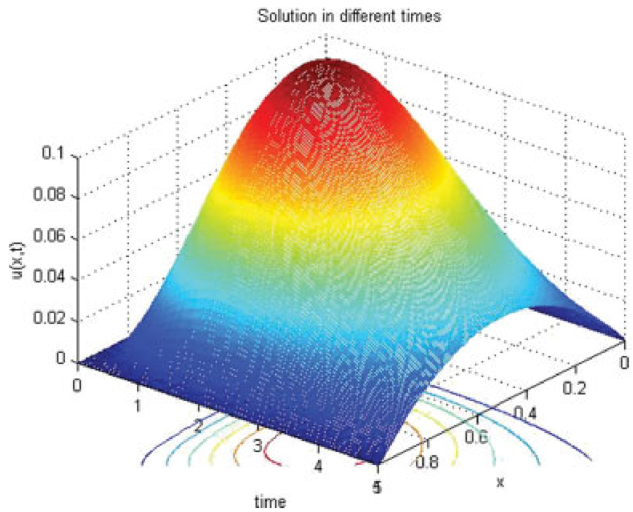

FIG. 6. Space-time graph of the solution up to $t = 5s$, with $dt = 0.001$ and $dx = 0.01$, for Example 3. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

TABLE V. Computational domain is [0, 1].

t	L_∞-error	L_2-error	RMS	Time(s)
1	8.5573×10^{-5}	6.1544×10^{-4}	6.1239×10^{-5}	0
3	6.4755×10^{-5}	4.6574×10^{-4}	4.6343×10^{-5}	1
5	5.7041×10^{-5}	4.1033×10^{-4}	4.0829×10^{-5}	2
7	5.4338×10^{-5}	3.9092×10^{-4}	3.8898×10^{-5}	3
10	5.2901×10^{-5}	3.8057×10^{-4}	3.7868×10^{-5}	5

L_∞, L_2, and RMS errors, with $dt = 0.001$, $dx = 0.01$.

D. Example 4

Similar to previous examples, we consider the hyperbolic telegraph Eq. (3.1) with $\alpha = 1$, $\beta = 1$, and $f(x,t) = x^2 + t - 1$ in the interval $0 \le x \le 1$. The initial conditions are given by

$$\begin{cases} u(x,0) = g_1(x) = x^2, & 0 \le x \le 1 \\ u_t(x,0) = g_2(x) = 1, & 0 \le x \le 1 \end{cases}, \tag{4.8}$$

and the exact solution is

$$u(x,t) = x^2 + t. \tag{4.9}$$

We extract the boundary function $h(x,t)$ from the exact solution. The L_∞ and L_2, errors and RMS of errors are obtained in Table V for $t = 1, 3, 5, 7$, and 10. The graph of analytical and estimated functions for $t = 10$ is given in Figure 7. We also draw the space-time graph of estimated solution in Figure 8.

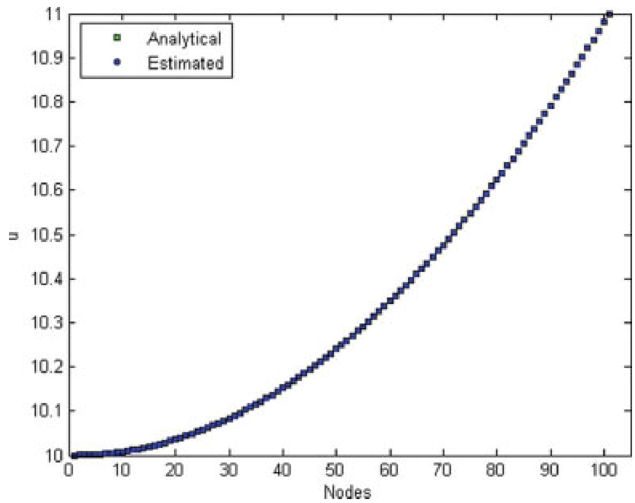

FIG. 7. Analytical and estimated function in $t = 10s$, with $dt = 0.001$ and $dx = 0.01$, for Example 4. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

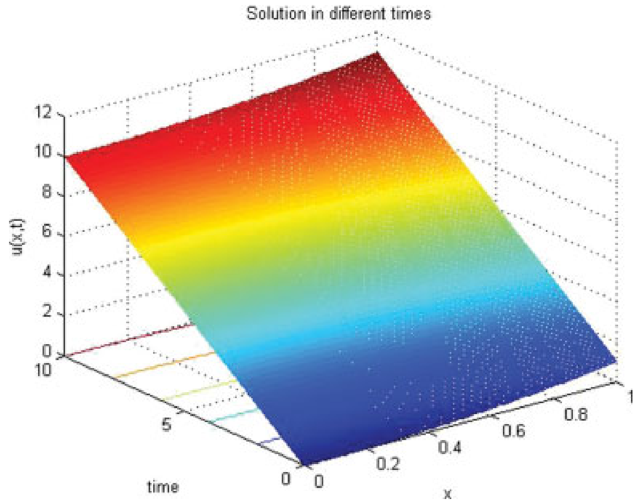

FIG. 8. Space-time graph of the solution up to $t = 10s$, with $dt = 0.001$ and $dx = 0.01$, for Example 4. [Color figure can be viewed in the online issue, which is available at www.interscience.wiley.com.]

V. CONCLUSION

In this article, we discussed on second-order hyperbolic telegraph equation. We proposed a numerical scheme to solve this hyperbolic equation using collocation points and approximating directly the solution using the TPS radial basis function. The scheme works in a similar fashion as finite difference methods [36]. The numerical results given in the previous section demonstrate the good accuracy of the scheme proposed in this research. The main advantage of the new method over finite difference techniques is that, latter methods provide the solution of the problem on mesh points only. The method proposed in this work can be extended to solve the important nonlinear partial differential equations investigated in [37–39].

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