

# A Survey of Graph-Based Resource Management in Wireless Networks - Part I: Optimization Approaches

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**Abstract**—The evolution of wireless communications and networking technologies has led significantly expansion of the dimensionality of network resources, which compels innovations in resource management. Graphs, a classic discrete mathematical tool, have long been widely used for resource management thanks to their capabilities to model complex relationships and interactions among elements in wireless networks. Recently, resource management over graphs embraces various advanced approaches of graph optimization and graph learning, aligned with evolving demands in future wireless networks. To better learn recent research landscape and explore important trends, this two-part survey provides a comprehensive overview for resource management via graph optimization and learning. Part I presents the fundamentals of graph optimization and provides a recent literature review of graph optimization for resource management in various wireless communication scenarios, including cellular networks, device-to-device communications, multi-hop networks, multi-antenna systems, edge caching and computing, and non-terrestrial networks. Part II gives the basics of graph learning and provides a state-of-the-art literature review of graph learning in wireless networks for addressing various resource management issues, covering power control, spectrum management, beamforming design, task scheduling, and aerial coverage planning. A discussion of technical challenges and future research directions is covered in Part II.

**Index Terms**—Wireless networks, resource management, graph

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## I. INTRODUCTION

Wireless communication and networking technologies have undergone rapid advancements in the past few decades, substantially augmenting available dimensions of network resources [1]. Developments in radio transmission techniques facilitate the adequate exploitation and unified scheduling of multi-dimensional resources in spatial, time, frequency, code, and power domains, which substantially enhances network capacity and connectivity. Furthermore, advancements in wireless networking techniques enable hybrid usage and planning of communication and computation resources, terrestrial and aerial resources, etc., which effectively improves network coverage and service provision. While all these technical innovations promote various performance indicators of wireless networks, they increase the difficulty and complexity of resource management [2]. Consequently, designing effective and efficient resource management schemes to adapt to the rapid technical transformation of wireless networks has attracted intensive research interest in both academia and industry.

Numerous theories have been applied to resource management in wireless network, such as optimization theory, queue theory, game theory, etc. Among them, graph theory has been used extensively over a long period to define and handle many different kinds of operation problems in wireless networks. Wireless nodes, such as devices and infrastructures, and their relationships, such as connection and interference, can be represented by vertices and edges in a graph, respectively. In this light, resource management problems can be modeled as different optimization problems over graphs, such as graph coloring, maximum flow, shortest path, etc. As a major branch of combinatorial optimization and discrete mathematics, graph optimization has developed many practical algorithms directly applicable for resource management. Recently, graph learning, also known as graph representation learning, has emerged as an effective approach in machine learning and has been applied to resource management in wireless networks. It is capable of capturing the structure and features of graph data to generate representation vectors to support resource management. Particularly, the graph neural network (GNN) is the latest research outcome in graph learning, which has been utilized to devise many resource management methods [3]. Compared to

traditional graph optimization and learning methods [4], [5], GNN-based methods offer superior scalability, generalization, and computational efficiency. Therefore, graph-based resource management remains a promising and attractive research field.

#### A. Background: From Graph Optimization to Graph Learning

Graph optimization has been widely employed in resource management of wireless networks, thanks to its adaptability and efficiency. As early as 1999, Chawla and Qiu adopted graph coloring to address beam switching issue in cellular networks for interference avoidance [6]. Since 2000, a large amount of research literature has applied various graph optimization approaches to resource management in wireless networks. Helmy proposed small world graphs for wireless networks to analyze and improve network connectivity [7]. Jain *et al.* [8] and Kodialam *et al.* [9] innovatively introduced the interference graph which is regarded as a foundational graph model for many optimization algorithms, such as graph coloring and maximum independent set, in wireless link scheduling and resource allocation. After 2010, advanced graph theoretical models and methods, such as hypergraphs [10]–[12], are utilized to model and depict wireless network for emerging network architecture and radio access technologies (RATs). Graph optimization for resource management features the following advantages.

- 1) *Adaptability*: The topology of wireless networks as well as relationships between network elements can be directly or easily represented as graph models to facilitate the implementation of graph optimization algorithms.
- 2) *Theoretical foundation*: Graph optimization has developed a lot of achievable algorithms which can balance the optimality and efficiency of dealing with various resource management problems.

However, graph optimization still faces challenges in managing multi-dimensional resources. First, as network scale increases, the size of graph model grows as well, which aggravates burdens of graph data storage and processing. Second, most of graph optimization problems are combinatorial optimization problems that usually cannot be solved in polynomial time. Hence, the overhead of graph optimization algorithms may not favor the timeliness of scheduling, especially for low-latency requirements of future wireless communications.

Recently, graph learning has been applied to resource management in wireless networks to enhance computational efficiency while maintaining optimality, where GNN is a prominent technique. Due to graph-in-graph-out architecture and message passing mechanism, GNN can extract useful information from topological structure and features of graph model to achieve problem solution. Eisen and Ribeiro first employed GNN in wireless networks to solve link scheduling problems in multi-hop networks and multiple access scheduling problems in cellular networks [13], [14]. Shen *et al.* demonstrated that GNNs converge faster and exhibit superior generalization in large-scale wireless networks compared to traditional deep neural networks, such as multi-layer perceptron [15]–[18]. Chowdhury *et al.* [19] and Yang *et al.* [20] integrated GNNs with existing iterative algorithms for power control, which

leverages the efficiency of GNNs and the accuracy of iterative algorithms at the same time. In recent years, many studies have combined GNNs with advanced learning frameworks, such as reinforcement learning (RL), aiming to adapt to dynamic changes and randomness in wireless networks [21], [22]. Apart from GNNs, deep learning-powered graph embedding techniques are used for access control and link scheduling in wireless networks to improve model generalization and training efficiency [23], [24]. Graph learning for resource management has the following advantages.

- 1) *Scalability*: Graph learning, particularly GNNs, can be applied to large-scale wireless networks because the number of parameters in GNN models is independent of network size.
- 2) *Efficiency*: GNNs have high training efficiency and can achieve the expected performance with a less number of network samples.
- 3) *Generalization*: GNNs offer good generalization capability for different network statuses and configurations, such as quality of service (QoS) settings, the number of users or channels, etc., due to their permutation-invariant property.
- 4) *Compatibility*: Graph learning methods can be easily implemented and fine-tuned on graph models constructed for resource management issues.

Graph learning-based resource management is still an emerging research direction, whose optimality and efficiency have substantial potential to be improved. Therefore, many related research topics have emerged, e.g., integrating domain knowledge from wireless communications and networking with GNN design.

#### B. Motivation and Contributions

There have been several outstanding surveys on graph optimization and learning for wireless communications and networking, which are summarized in Table I. First category of these surveys focused on the application of graph optimization for wireless networks. Cardieri comprehensively reviewed graph optimization approaches for interference modeling in wireless ad hoc networks and emphasized its application domains [25]. Pathak *et al.* thoroughly investigated graph optimization for cross-layer designs in wireless mesh networks [26]. Majeed *et al.* surveyed the application of graph theory to model various issues in computer networks including Internet of Things (IoT), web page ranking, network topology generation, and encryption [27]. Second category of the literature is dedicated to investigating graph learning approaches for wireless communications and networking. He *et al.* presented several applications of GNNs to resource allocation in wireless networks as well as other emerging problems such as channel estimation and traffic prediction [28]. Jiang comprehensively reviewed diverse GNN models applied to operation management and optimization of communication networks involving both wired and wireless scenarios [29]. Tam *et al.* mainly reviewed the application of GNNs to network management of core systems and networks, especially for software defined networking (SDN) control and network

Table I  
COMPARISON WITH SELECTED SURVEYS.

References	Methodology <sup>†</sup>		Networks	Subjects	Contributions
	GO	GL			
[25]	✓		Wireless ad hoc networks	Interference modeling	This paper surveyed graph optimization for interference modeling in wireless ad hoc networks, emphasizing its application domains and illustrated with examples.
[26]	✓		Wireless mesh networks	Cross-layer designs	This paper surveyed fundamental design problems in wireless mesh networks and their joint designs, where graph optimization is a crucial methodology.
[27]	✓		Computer networks	Network modeling	This survey reviewed the application of graph theory for computer networks including IoT, web page ranking, network topology generation, and encryption.
[28]		✓	Various wireless networks	Resource management	This work reviewed the application of GNNs to resource allocation in various wireless networks, i.e., mesh/ad hoc networks, cellular networks and WLAN, as well as several other issues, such as channel estimation and traffic prediction.
[29]		✓	Wired and wireless networks	Network management	This paper surveyed different GNN models applied to network operation management and optimization in wired and wireless networks as well as SDN.
[30]		✓	Core networks	SDN and NFV optimization	This work reviewed GNN for intelligent network management and orchestration to optimize control policies in SDN and NFV enabled core networks for wired and wireless communications.
[31]		✓	IoT	Network security and management	This survey reviewed related research and summarized the progress of using graph learning to network anomaly detection, malware detection, IoT device and service management, etc.
[32]		✓	IoT	Sensing application	This survey presented a deep dive analysis of GNN designs in various IoT sensing scenarios and an overarching list of public data and source codes.
[33]	✓	✓	NTN	Resource management	This paper reviewed resource allocation methods based on both graph optimization and graph learning, and proposed a graph-based resource management framework for NTN and its integration with terrestrial networks.
Ours	✓	✓	Various wireless networks	Resource management	A survey comprehensively and systematically reviewed resource management issues and solutions from the perspectives of both graph optimization and graph learning in various advanced wireless networks.

<sup>†</sup>GO: Graph optimization. GL: Graph learning.

function virtualization (NFV) orchestration [30]. Li *et al.* surveyed graph learning methods for network security and management in IoT scenarios [31]. Dong *et al.* presented a comprehensive overview of GNN applications in various IoT sensing environments with a list of public data and source codes [32]. In addition to the above two categories, Ivanov *et al.* reviewed resource allocation methods based on the graph optimization and learning from perspective current wireless networks and future non-terrestrial networks (NTNs) [33].

In summary, the majority of existing surveys have focused exclusively on either graph optimization or graph learning for wireless communications and networking. Although some literature provides an overview of both graph optimization and graph learning, the discussed wireless communication scenarios and network types are often incomplete, particularly for emerging cellular and cell-free networks, edge caching and edge computing. Additionally, few surveys specifically address resource management issues using graph optimization and learning. These gaps motivate us to conduct a comprehensive and systematic literature review of the latest resource management techniques over graphs, considering both perspectives of graph optimization and graph learning. We summarize the contributions of this two-part survey as follows.

- We introduce key fundamentals of graph theory and graph optimization problems with typical algorithms in Part I as well as basics of graph learning and several modern GNN models in Part II. This demonstrates how knowledge of graph optimization lays the foundation for implementing graph learning.
- We categorize and discuss graph optimization approaches for resource management across different scenarios of wireless communications in Part I. In each scenario, typical resource management issues are distinctly presented, meanwhile the literature of practical graph optimization algorithms for each issue is systematically reviewed.
- We classify and review the application of graph learning methods according to different resource management issues in Part II. In this way, the characteristics and components of each issue is demonstrated and the applicable graph learning approaches in the literature are comprehensively reviewed.
- We summarize technical challenges and future directions of graph optimization and learning methods for resource management in Part II. These challenges are primarily centered around new features brought by the evolution of wireless networks. Future directions align with the

development of advanced graph optimization and learning techniques.

### C. Paper Organization

Fig. 1 shows the organization of the remainder of this two-part survey which comes in two parts. In Part I, Section II first presents the basics of graph theory and various graph optimization problems along with their relevant algorithms. Section III then provides a recent literature review of graph optimization for resource management in wireless networks, categorized by different scenarios including cellular networks, device-to-device (D2D) communications, multi-hop networks, multi-antenna systems, edge caching and computing, and NTN. Part II introduces the fundamentals of graph learning and provides a state-of-the-art literature review of graph learning approaches for resource management in wireless networks. Furthermore, a discussion of technical challenges and future directions in this field is presented in Part II.

## II. FUNDAMENTALS OF GRAPH OPTIMIZATION AND GRAPH LEARNING

In this section, we first provide the basic knowledge of graph theory. Then, several graph optimization problems and their methods are presented.

### A. Basics of Graph

In mathematics, a graph is defined by a pair  $G = (\mathcal{V}, \mathcal{E})$  [34].  $\mathcal{V} = \mathcal{V}(G) = \{v_1, v_2, \dots, v_m, \dots\}$  is a vertex set where the elements are called vertices representing objects in a graph.  $\mathcal{E} = \mathcal{E}(G) = \{e_1, e_2, \dots, e_n, \dots\}$  is an edge set where the elements are called edges representing relationships between vertices. Fig. 2a illustrates an example of a graph. If  $v_i$  and  $v_j$  are the endpoints of an edge  $e_n$ ,  $e_n$  is incident on  $v_i$  and on  $v_j$  meanwhile  $v_i$  and  $v_j$  are adjacent. The loop is a special edge whose two endpoints are one vertex. All the vertices adjacent to  $v_i$  are called its neighbours  $\mathcal{N}_G(v_i)$ . If graphs  $G$  and  $H$  meet  $\mathcal{V}(H) \subseteq \mathcal{V}(G)$  and  $\mathcal{E}(H) \subseteq \mathcal{E}(G)$ ,  $H$  is a subgraph of  $G$ , i.e.,  $H \subseteq G$ . In particular,  $H = G$  if  $\mathcal{V}(H) = \mathcal{V}(G)$  and  $\mathcal{E}(H) = \mathcal{E}(G)$ .

1) *Numeric*: There are two fundamentally numerical values in a graph, i.e., the degree and the weight. The degree of a vertex  $d_G(v_m)$  denotes the number of edges connecting this vertex. As per Fig. 2a,  $d_G(v_1) = 2$  because there are 2 edges connecting  $v_1$  to  $v_2$  and to  $v_3$ , respectively. The weight can be associated with either a vertex or an edge. In this case, the graph is called a weighted graph. The weight of an edge  $w_G(e_n)$  is often referred to as the cost of the edge, such as the distance of a path, the length of a link, the capacity of a channel, etc. The weight of a vertex  $w_G(v_m)$  is used to measure a cost of the vertex, such as the priority of a user, the data stored by a device, the transmit power of a node, etc.

2) *Direction*: In a graph  $G$ , the edge set  $\mathcal{E}$  consists of either undirected edges or directed edges. If all the elements in  $\mathcal{E}$  are undirected edges, the graph is called undirected graph. If all the elements in  $\mathcal{E}$  are directed edges, the graph is called directed graph. Figs. 2a and 2b illustrate examples of undirected graph and directed graph, respectively.

In an undirected graph, an edge  $e_n$  connects the unordered pair of vertices, e.g.,  $v_i$  and  $v_j$ , which is expressed as  $e_n = v_i v_j$ . Hence, the relationship of vertices connected by one edge is symmetric. The bipartite graph is a special type of undirected graph which consists of two disjoint vertex subsets and there is not any edge connecting vertices in the same vertex subset. Fig. 2c shows an example of bipartite graph. In a directed graph, each edge, also called the arc, has a direction with an arrow. A directed edge  $e_n$  is expressed as  $e_n = (v_i, v_j)$  from  $v_i$  to  $v_j$ , e.g.,  $e_1 = (v_1, v_2)$  in Fig. 2b. Thereby,  $e_n$  is the out-arc of  $v_i$  and the in-arc of  $v_j$ .  $v_i$  is called in-neighbour of  $v_j$ .  $v_j$  is called out-neighbour of  $v_i$ .  $\mathcal{N}_G^+(v_m)$  and  $\mathcal{N}_G^-(v_m)$  are out-neighbour set and in-neighbour set of  $v$ , respectively.

3) *Representations*: There are several approaches to represent a graph. The most straightforward representation approach is the diagram form, as per Fig. 2. In order to facilitate mathematical operations and storage, the matrix has become an efficient and common form for graph representation.

- *Incidence matrix*:  $\mathbf{I}(G) = [i_{n,m}]$ ,  $v_m \in \mathcal{V}$ ,  $e_n \in \mathcal{E}$ , is a  $|\mathcal{E}| \times |\mathcal{V}|$  matrix which encodes the relations of vertices and edges in  $G = (\mathcal{V}, \mathcal{E})$  without loops.  $|\cdot|$  expresses the cardinality of a set, i.e., the number of elements in the set. For an undirected graph,  $i_{n,m} = 1$  if vertex  $v_m$  is incident with edge  $e_n$ , otherwise  $i_{n,m} = 0$ . For a directed graph,  $i_{n,m} = 1$  if vertex  $v_m$  is the head of edge  $e_n$ .  $i_{n,m} = -1$  if vertex  $v_m$  is the tail of edge  $e_n$ . Otherwise,  $i_{n,m} = 0$ .
- *Adjacency matrix*:  $\mathbf{A}(G) = [a_{i,j}]$  for  $G = (\mathcal{V}, \mathcal{E})$  is a square matrix of order  $|\mathcal{V}|$  where each element indicates the adjacency relation between a pair of vertices. For an undirected graph,  $a_{i,j}$  is equal to the number of edges between vertices  $v_i$  and  $v_j$ . For a directed graph,  $a_{i,j}$  is equal to the number of edges directed from  $v_i$  to  $v_j$ . Besides, the weight matrix is an extension of the adjacency matrix to represent the edge-weighted graph without multiple arcs and edges.
- *Weight matrix*:  $\mathbf{W}(G) = [w_{i,j}]$  is an extension of the adjacency matrix and represents the edge-weighted graph without multiple arcs and edges. In a weight matrix,  $w_{i,j} = w_G(e_n)$  where  $e_n$  is an existing edge or arc between vertices  $v_i$  and  $v_j$ . If  $i = j$ ,  $w_{i,j} = L$ . Otherwise,  $w_{i,j} = K$ .  $L$  and  $K$  are definable values and equal to  $\infty$ ,  $-\infty$ , 0, etc., according to the actual requirements.

4) *Hypergraph*: Hypergraphs are a generalization of a graph where an edge joins any number of vertices instead of at most two vertices in the ordinary graph. The edge in hypergraphs is called hyperedge. Each hyperedge is a non-empty subset of vertices. The number of vertices is called the order of the hypergraph. The number of hyperedges is called the size of the hypergraph. An undirected hypergraph  $H$  is expressed as  $H = (\mathcal{X}, \mathcal{E})$ , where  $\mathcal{X}$  is a set of vertices and  $\mathcal{E}$  is a set of hyperedge. Fig. 2d shows an example of an undirected hypergraph. A directed hypergraph contains the hyperedge set  $\mathcal{E}$  where each hyperedge is an ordered pair of subsets of  $\mathcal{X}$ . Incidence matrix and adjacency matrix are common representation matrices for hypergraphs.

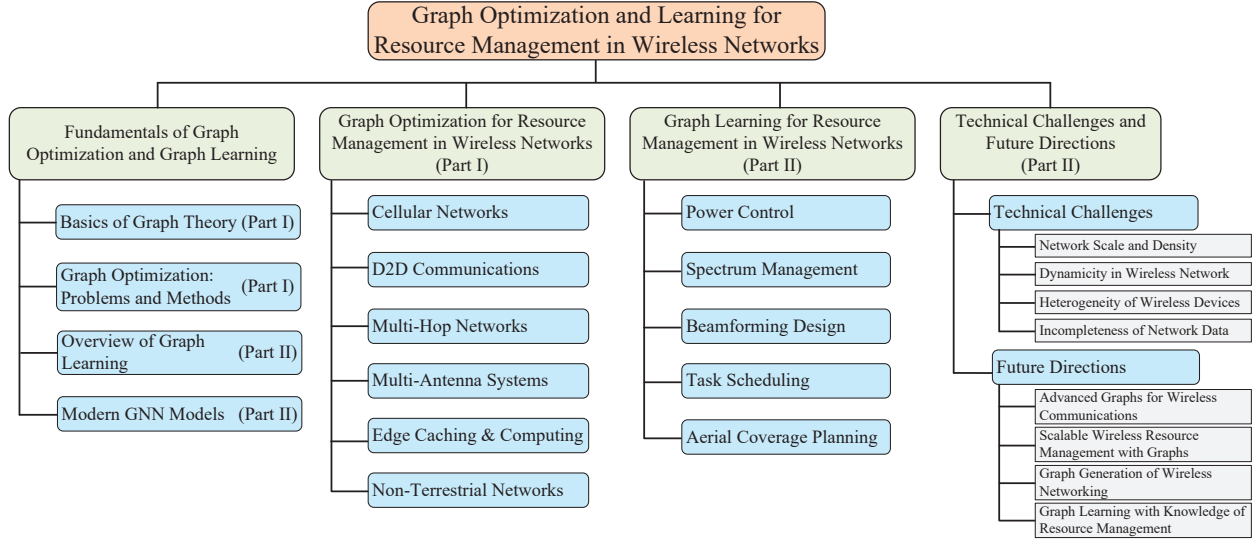


Figure 1. Organization of this paper and an overview of major topics.

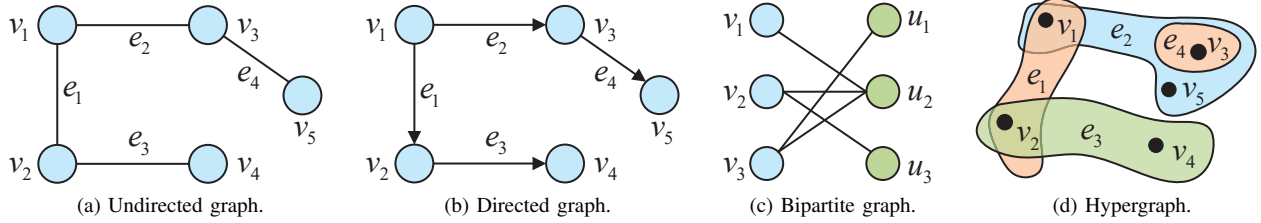


Figure 2. Examples of graphs.

### B. Graph Optimization: Problems and Methods

Graph optimization, as a primary branch of combinatorial optimization, uses the graph to model optimization problems and utilizes the characteristics of constructed graph to design corresponding solutions and algorithms. In this subsection, we introduce several graph optimization problems and methods applicable in wireless communications and networking.

1) *Graph Coloring*: It is essentially a generalization of assignment problem. It aims to assign colors to vertices or edges in an undirected graph so that no two adjacent vertices or edges are of the same color. For example, Fig. 2c is stained with two colors. Colors can be used to represent resources in wireless networks. Taking vertex coloring as examples, three graph coloring problems are introduced as follows.

- *K-coloring judgment*: It is to judge whether one undirected graph  $G$  can be completely painted by given at most  $k$  colors.  $k$  is an integer.  $G$  is  $k$ -colorable, also called a  $k$ -coloring, if it can be painted by  $k$  colors. Existing  $k$ -coloring algorithms include Grover's algorithm [35], DSatur algorithm [36], etc.
- *Chromatic number*: As one of NP-complete problems, it is to find the minimum chromatic number of an undirected graph. Various algorithms based on backtracking and recurrence are developed with exponential computational complexity. Moreover, many greedy and heuristic algorithms are proposed, such as Welsh–Powell algorithm [37], Brélaz's heuristic algorithm [38].

- *Greedy coloring*: It considers vertices in a given order and in order assign each vertex with the smallest available color not used by its neighbours, appending a new color if required [39]. Different from  $k$ -coloring algorithms, the greedy coloring is not given the number of available colors.

2) *Shortest Path*: This problem aims to find a path between two vertices which has the minimum sum of edge weights. The shortest path problem can be defined over an undirected graph or a directed graph. A path in an undirected graph is a sequence of vertices, e.g.  $v_1 - v_2 - v_4$  in Fig. 2a. A path in a directed graph is a lineup of consecutive vertices connected by corresponding directed edges, e.g.  $v_1 \rightarrow v_3 \rightarrow v_5$  in Fig. 2b. There are three classic shortest path algorithms, i.e., Dijkstra's algorithm, Bellman-Ford algorithm, and Floyd–Warshall algorithm. Based on these three algorithms, many advanced algorithms are proposed, such as goal-directed algorithm and contraction hierarchies algorithm [40].

3) *Flow Network*: A flow network is a directed graph  $D = (\mathcal{V}, \mathcal{E})$  where there exist two special vertices of the source and the sink. Each arc  $e_n$  in a flow network has a capacity  $c(e_n)$  and a flow  $f(e_n)$  which are non-negative reals. The source only has the outgoing flow. The sink only has the incoming flow. Except for the source and the sink, the amount of a flow into each vertex must equals that out of it. A flow network can be defined by a tuple  $N = (D, \mathbf{c}, v_s, v_k)$  where  $v_s$  and  $v_k$  represents the source and the sink, respectively.  $\mathbf{c}$  is a vector

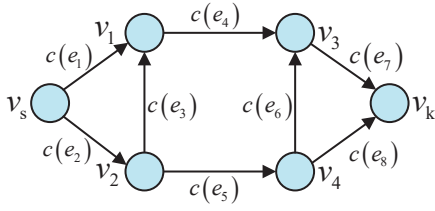


Figure 3. An example of flow network.

including the capacity of each edge. In general, a flow network does not include multiple arcs. Most of flow networks can be formulated by the integral linear programming problem. Fig. 3 illustrates an example of flow network. There are two typical problems for flow network.

- *Maximum flow*: It aims at finding the maximum acceptable flow from the source to the sink. The *max-flow min-cut* theorem, a well-known theorem in the flow network, states the maximum amount of flow passing through the source to the sink is equivalent to the sum weight of edges in a minimum cut. The minimum cut is defined as the smallest sum weight of edges which can disconnect the source and the sink if removed. Based on the max-flow min-cut theorem, many efficient optimization algorithms are proposed such as Ford-Fulkerson algorithm, Edmonds-Karp algorithm, Dinic's algorithm [41].
- *Minimum-cost flow*: It aims at finding the lowest possible price to send a certain amount of flow from the source to the sink. Besides  $c(e_n)$  and  $f(e_n)$ , each arc has a specific weight  $u(e_n)$  representing the cost per unit of flow. The cost of a flow along  $e_n$  equals  $f(e_n) \cdot u(e_n)$ . There are many efficient algorithms based integral linear programming for solving this problem [42].

In addition, there are derivative problems in the flow network, i.e., double-capacity flow problem, multi-source (or sink) flow problem, etc., some of which are still open problems.

4) *Bipartite Matching*: In a bipartite graph, the bipartite matching, also called two-sided matching, is to find a subset of edges where any two of edges do not have the same vertex. Obtained edge subset is called a matching. If a matching covers all the vertices, it is a perfect matching. Maximal matching, maximum-weight matching and stable matching are representative bipartite matching problems.

- *Maximal matching*: The maximal matching is to find a matching including edges as many as possible. If the matching contains the largest number of edges, it is a *maximum matching*. The Hopcroft-Karp algorithm is an efficient solution for this problem [43].
- *Maximum-weight matching*: In a weighted bipartite graph, it aims to find a matching in which the sum weight of edges is maximized. The Hungarian algorithm, also known as the Kuhn-Munkres algorithm, is the best-known algorithm for solving this problem [43].
- *Stable matching*: In this problem, each vertex has an ordering of preference for vertices in the opposite side. A matching is stable if there is not any pair of vertices that both prefer each other to their current partner under the

matching. The Gale-Shapley algorithm is well-known to find the one-to-one stable matching [43]. Matching game theory is efficient to find a stable result in *many-to-one matching* and *many-to-many matching* where each vertex is allowed to have two or more partners [44].

5) *Independent Set and Clique*: The independent set and the clique are complementary. An independent set is a vertex subset in an undirected graph, any two of which are not adjacent. In contrast, a clique is a vertex set where any two vertices are adjacent. Actually, the graph coloring is to partition vertices into different independent sets. Taking independent set as example, there are two typical problems as follows.

- *Maximal independent set*: It aims to find an independent set including vertices as many as possible. If the independent set includes the largest number of vertices, it is a *maximum independent set*. For example,  $\{v_1, v_4, v_5\}$  is a maximum independent set in Fig. 2a. As a NP-hard problem, its optimal solution can be achieved by the brute force algorithm. There are greedy solutions such as Luby's algorithm and Blleloch's algorithm [45].
- *Maximum-weight independent set (MWIS)*: It aims to find an independent set in which the sum weight of vertices is maximized. There are customized branch-and-bound (BnB) approaches and greedy algorithms proposed for solving this problem [46].

Due to the complementarity, the *maximal clique* and the *maximum-weight clique* are defined to find a clique as large as possible and a clique with maximum sum weight, respectively. If a clique contains the largest number of vertices, it is a *maximum clique*. The problem solutions about cliques are compatible with corresponding problems for independent sets.

### III. GRAPH OPTIMIZATION FOR RESOURCE MANAGEMENT IN WIRELESS NETWORKS

Many aspects of wireless networks can be modeled by graphs due to their powerful representation ability. For instance, the network topology can be represented as an undirected graph. In this graph, each vertex represents a network node or a communication link, while each edge represents the connection or interference between vertices [47]. On this basis, different colors can be used to represent available wireless channels to be assigned to different vertices [6]. Consequently, different graphs can be constructed to serve different motivations and objectives. Suitable graph optimization methods are then employed to solve corresponding problems on these constructed graphs. This section provides a review of the application of graph optimization for resource management in the following scenarios of wireless networking.

- *Cellular networks*: The base station (BS) is a vital network infrastructure in cellular communications to provide a cell with the network coverage. Each user needs to associate with at least one BS to access the network. A variety of graph optimization approaches are used to formulate and solve resource management issues in cellular networks.

- *D2D Communication*: The proximity service enables two or more users to communicate with each other without the assist of BSs, which is called D2D communication technique. Graph optimization can be used as an effective tool to schedule D2D communications.
- *Multi-hop networks*: A multi-hop network comprises a group of nodes able to communicate with or relay for each other. Multi-hop networks serve as a crucial foundation for implementing graph optimization methods in resource management.
- *Multi-antenna systems*: In multi-antenna systems, the transmitter and/or the receiver is equipped with the multi-antenna array to form new transmission dimensions for increasing link capacity. Graph optimization is applied to channel and pilot allocation in multi-antenna systems.
- *Edge caching and computing*: The computation and storage resources at the edge of wireless networks are as important as the communication resources, which motivates emerging applications and use cases. Recently, various graph optimization approaches are employed to tackle resource management issues in edge caching and computing.
- *NTNs*: Satellites and aerial infrastructures play primary roles in NTN. There are many novel resource management issues in NTN and their integration with terrestrial networks. Graph optimization is mainly used to link scheduling and resource allocation in NTN.

All the above six scenarios cover almost all primary use cases in current and future wireless networks. Meanwhile, graph optimization has been widely and effectively applied to resource management in these six scenarios.

#### A. Cellular Networks

Cellular networks are currently the most dominant wireless networking technology. Graph optimization has been applied to resource management in cellular networks for a long time. In the early works, a max  $k$ -cut based resource allocation algorithm is designed for a multi-cell downlink orthogonal frequency division multiple access (OFDMA) network [48]. The maximal matching over random bipartite graph is used for subcarrier assignment in a single-cell OFDMA network [49], [50]. The minimum-cost flow is applied to resource allocation for a frame-based OFDMA network with the consideration of QoS [51]. This subsection focuses on research efforts over the past decade and review recent literature on resource management in single-cell networks and multi-cell networks, respectively, with different RATs.

1) *Single-Cell Networks*: Graph optimization is mainly used for channel allocation to enhance spectrum efficiency of single-cell networks that typically consist of one BS and multiple users, as per Fig. 4.

For orthogonal multiple access (OMA) networks where each channel is only assigned by at most one user, a graph labeling algorithm is designed for consecutive-block channel allocation in an uplink single-carrier frequency division multiple access (SC-FDMA) system, in the graph underlying which each vertex represents a user and each edge represents a channel block

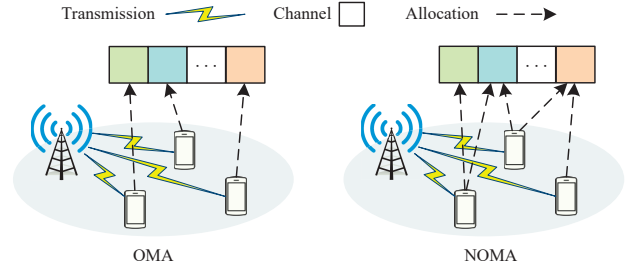


Figure 4. Channel allocation in the single-cell network.

associated with multiple weights to specify the performance metric, i.e., utility, power, or the number of channels. This algorithm is a variant of graph coloring and can achieve the near-optimal solution [52]. A maximal matching algorithm is applied to channel allocation in a downlink OFDMA system that is modeled as a multi-queue system with as many servers as the number of frequency channels. A random bipartite graph is exploited to formulate queue lengths, traffic arrival, and other external randomness of users as well as matching relationship between user vertices and channel vertices [53]. Moreover, bipartite matching algorithms are also performed to tackle channel allocation in single-cell networks with other specific OMA techniques and applied scenarios [54]–[56].

For non-orthogonal multiple access (NOMA), matching game theory is usually used for channel allocation. Since each channel is allowed to be reused by multiple users, channel allocation problems in NOMA systems can be formulated as many-to-one matching or many-to-many matching over bipartite graphs. A many-to-many matching algorithm is proposed for channel allocation in a downlink single-cell NOMA network, which can achieve the maximum network capacity with sufficiently large iterations [57]. In the same scenario, the quality of service for users is further considered in the process of many-to-many matching [58]. A many-to-one matching algorithm is designed for channel allocation in uplink single-cell NOMA network, which can converge to stable matching with limited iterations [59]. Besides matching game theory, an MWIS based algorithm is proposed for channel allocation in an uplink single-cell NOMA network to maximize network capacity. In its graph, each vertex represents a combination of two users and one channel and each edge connects two vertices including the same user or channel [60]. Based on this work, a maximum independent set based algorithm is further designed to jointly optimize access control and channel allocation [61].

Recently, graph optimization is applied to channel allocation in emerging mobile use cases and technologies. The maximum-weight matching is exploited to propose channel allocation and sharing scheme for ultra-reliable low latency communications (uRLLC) in a single-cell IoT network to increase spectrum efficiency [62], [63]. Greedy coloring is utilized for channel allocation and user scheduling in single-cell networks with in-band full-duplex (IBFD) technology to maximize spectrum efficiency and promote frequency sharing [64], [65].

2) *Multi-Cell Networks*: Different from single-cell networks, inter-cell interference is a dominating challenge for

resource management of multi-cell networks that include multiple BSs and multiple users. The interference graph is an explicit tool to characterize interference among users or cells, in which each vertex represents a user or a BS and each edge connects two vertices strongly interfering with each other.

By means of interference graph, several low-complexity heuristic algorithms are proposed to allocate frequency channels for inter-cell interference mitigation [66], [67]. Game-theoretic approaches are also proposed to operate on interference graph for user scheduling and channel allocation to restrain interference multi-cell networks [68], [69]. To accelerate the implementation of interference graph, a machine learning-based graph construction method is proposed to improve the accuracy and practicability [70]. It is worth noting that due to good compatibility, greedy coloring is popularly applied on interference graph to interference mitigation in wireless networks [71], as per Fig. 5. A partially-distributed resource allocation algorithm is proposed to apply greedy coloring for spectrum channel allocation among small cells [72]. For moving small-cell networks, the time interval dependent interference graph is exploited to design a greedy coloring based resource block (RB) allocation algorithm for alleviating time-varying interference [73]. To further mitigate inter-cell interference, a modified  $k$ -coloring algorithm is designed for channel allocation in interference alignment (IA) enabled OFDMA multi-cell networks [74]. A joint IA and subchannel allocation scheme is further proposed which utilizes a greedy  $k$ -coloring algorithm to find the smallest number of subchannels required [75].

Various graph optimization methods have been applied for resource management in multi-cell networks except for interference graph. A minimum-cost flow algorithm is developed to switch on/off BSs dynamically in multi-cell networks for energy saving [76]. The maximum independent set is exploited to formulate link scheduling problem and propose a computationally efficient algorithm for a two-tone spectrum-sharing heterogeneous cellular network (HetNet) [77]. A maximum-weight clique-based algorithm is proposed for joint link scheduling and power control in a cloud-radio access network (C-RAN), which can find optimal solution with low complexity [78]. The bipartite matching is as well utilized to manage spectrum resource in multi-cell networks [79]–[81]. A bipartite stable matching based network selection algorithm is designed to optimize overall quality of experience of users under fairness assurance in an ultra-dense HetNet [82]. A maximum matching-based subchannel allocation algorithm is proposed for non-coherent joint transmission to restrain multi-cell interference [83]. Furthermore, hypergraph is utilized to design resource allocation algorithms for multi-cell networks with advanced RATs and application scenarios. An interference hypergraph is established to design a greedy spectrum resource allocation algorithm in a NOMA-enabled dense HetNet, where each vertex represents the usage of a subchannel by an user pair and each hyperedge contains vertices corresponding to the same user pair [84]. A hypergraph-based maximum-weight clique method is proposed for channel allocation to improve spectrum efficiency in a NOMA-based industrial IoT network [85].

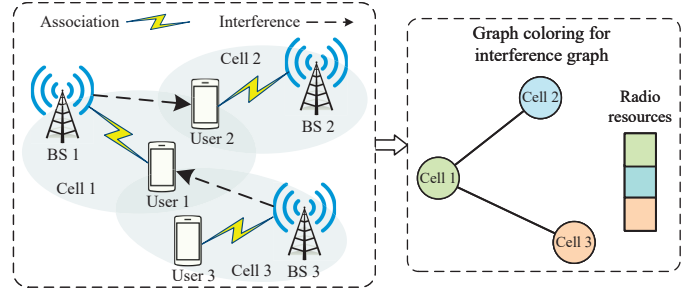


Figure 5. Inter-cell interference coordination by graph coloring

*Lessons learned 1:* Graph optimization is an effective and long-standing theoretical tool for resource management in cellular networks. In single-cell networks, graph optimization approaches are mainly used for channel allocation to enhance spectrum efficiency. Among them, graph coloring and bipartite matching are two common methods. Results in literature show that a graph coloring-based algorithm can achieve a near-optimal solution in an uplink OMA single-cell network. For NOMA single-cell networks, matching game theory and independent set-based algorithms are effective for channel reuse to promote resource utilization. In multi-cell networks, interference coordination is the primary challenge of resource management. To tackle this challenge, various interference graph-based algorithms are proposed. Greedy coloring is successfully applied spectrum channel allocation among small cells. Furthermore, minimum-cost flow, independent set and clique-based algorithms, bipartite matching, and hypergraph are utilized for different resource management problems to mitigate interference and improve resource utilization. Table II summarizes the reviewed resource management approaches using graph optimization in cellular networks along with references. From the literature review, we can see that graph optimization is promised to be applied to resource management in future cellular networks, such as cell-free networks, dense heterogeneous cellular networks, etc.

## B. D2D Communication

As a complementary technique, D2D communication enables direct communication between two mobile users in close proximity without going through cellular BS or core network [86], [87]. There are two typical working modes that are underlay mode and overlay mode, as per Fig. 6. In underlay mode, D2D communication reuses cellular frequency resource to improve spectrum efficiency yet causing cross-tier interference between cellular links and D2D pairs. In overlay mode, D2D communication is not allowed to use cellular frequency resource and only uses a dedicated frequency band. Note that resource management of D2D communication is controlled by cellular networks regardless of working modes. Therefore, cross-tier interference and spectrum competition become more severe in D2D and cellular hybrid networks compared to cellular networks. This subsection reviews the research literature on the application of graph optimization to D2D communications in different working modes.



Table II  
A SUMMARY OF RESOURCE MANAGEMENT APPROACHES USING GRAPH OPTIMIZATION IN CELLULAR NETWORKS

Networks	References	Methods	Graph Types	Issues	RATs
Single-Cell Networks	[52]	Labeling	Conflict graph	Channel allocation	SC-FDMA
	[53]	Maximal matching	Bipartite graph	Channel allocation	OFDMA
	[54]	Maximum-weight matching	Bipartite graph	Channel allocation	OFDM-IDMA
	[55], [56]	Maximum-weight matching	Bipartite graph	Channel allocation	OFDMA
	[57], [58]	Many-to-many matching	Bipartite graph	Channel allocation	NOMA
	[59]	Many-to-one matching	Bipartite graph	Channel allocation	NOMA
	[60]	MWIS	Conflict graph	Channel allocation	NOMA
	[61]	Maximum independent set	Conflict graph	Access control and channel allocation	NOMA
	[62], [63]	Maximum-weight matching	Bipartite graph	Channel allocation and sharing	OFDMA for uRLLC
	[64], [65]	Greedy coloring	Conflict graph	User scheduling and channel allocation	OMA with IBFD
Multi-Cell Networks	[66]	Heuristic	Interference graph	Channel allocation	OFDMA
	[67]	Heuristic	Interference graph	Channel allocation	TDMA
	[71]	Greedy coloring	Interference graph	Channel allocation	OMA
	[72]	Greedy coloring	Interference graph	Channel allocation	OFDMA
	[73]	Greedy coloring	Time interval dependent interference graph	RB allocation	OFDMA
	[74]	$k$ -coloring	Interference graph	Channel allocation	OFDMA with IA
	[75]	$k$ -coloring	Interference graph	IA and channel allocation	OFDMA
	[76]	Minimum-cost flow	Flow network	BS on/off switching	OFDMA
	[77]	Maximum independent set	Conflict graph	Link scheduling	FDMA
	[78]	Maximum-weight clique	Conflict graph	Link scheduling and power control	OFDMA
	[79]	Maximum-weight matching	Bipartite graph	Channel and power allocation	Spectrum aggregation
	[80]	Stable matching	Bipartite graph	Spectrum allocation	OMA
	[81]	Maximum-weight matching	Bipartite graph <td User scheduling	FDMA	
	[82]	Stable matching	Bipartite graph	Network selection	Hybrid access
	[83]	Maximum matching	Bipartite graph	Channel allocation	Non-coherent joint transmission
	[84]	Heuristic	Interference hypergraph	Spectrum channel allocation	NOMA
	[85]	Maximum-weight clique	Hypergraph	Channel allocation	NOMA

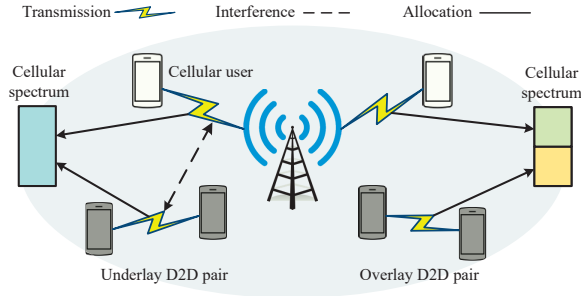


Figure 6. D2D communications in underlay and overlay mode.

1) *Underlay D2D Communication:* Graph optimization focuses on spectrum reuse among underlay D2D pairs and cellular links. The spectrum reuse between one cellular user and one D2D pair is first studied. Supposing that each cellular user has been assigned to orthogonal spectrum channel, a maximum-weight bipartite matching based scheme is proposed to select a suitable cellular user as an optimal reuse partner for each admissible D2D pair to maximize network capacity [88], as per Fig. 7. The conflict graph is used to propose a heuristic algorithm to match each cellular user's codebook to one

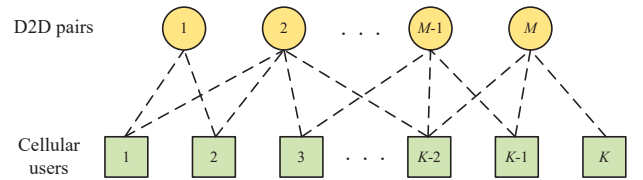


Figure 7. Bipartite matching between underlay D2D pairs and cellular users.

D2D pair in a D2D underlaying cellular network with sparse code multiple access (SCMA) that is an emerging NOMA technique [89]. In a NOMA-based D2D underlaying multi-cell network, a hypergraph greedy coloring based channel reuse algorithm is designed where the colors correspond to available channels and each hyperedge consists of cellular links and D2D pairs with a certain level of mutual interference [90]. Supposing that spectrum channels have not been assigned yet, a hypergraph based BnB algorithm is developed to obtain the optimal channel allocation and reuse in a D2D underlaying cellular network [91].

Second, one-to-many spectrum reuse between cellular links and D2D pairs is studied. A min-cut based transmission-direction optimization scheme over interference graph is de-

veloped to minimize total interference strength in a single-channel D2D underlaying cellular network [92]. A minimum-weight  $k$ -cut based spectrum reuse algorithm is proposed to assign exactly one cellular link to each cluster of D2D pairs to alleviate cross-tier interference [93]. A hypergraph greedy coloring based channel allocation algorithm is developed for both D2D pairs and cellular links to maximize network capacity with low complexity, which is shown to achieve a near-optimal performance [94]. Considering social ties among users, a social-aware resource allocation scheme is proposed which uses many-to-one matching to assign D2D pairs of each community with cellular spectrum resources of one other community [95].

Finally, many-to-many spectrum reuse between cellular links and D2D pairs is studied. To balance the effectiveness and the complexity, a  $k$ -coloring based spectrum resource sharing algorithm is proposed over interference graph for a D2D underlaying full-duplex cellular network to maximize network capacity [96].

2) *Overlay D2D Communication*: Graph optimization is usually applied to deal with resource allocation issues in overlay D2D communications. Due to dedicated spectrum resources, there is not any conflict between D2D pairs and cellular links. Hence, a system consisting of multiple overlay D2D pairs is also referred as a D2D network. A bipartite stable matching based spectrum reuse algorithm is proposed to match each secondary D2D pair to one primary D2D pair for spectrum utilization enhancement in a cognitive radio (CR)-assisted D2D network [97]. The bipartite stable matching is further used to group several cooperative users with social ties for data dissemination via D2D communications [98]. A graph based heuristic frequency assignment and duplex mode selection scheme is designed in a full-duplex D2D network to improve spectrum efficiency [99]. A completion time minimization algorithm is proposed for a D2D-aided caching fog radio access network (F-RAN), which uses the maximum-weight clique to minimize the possible completion time in downlink transmission and uses the maximum independent set to maximize the number of active users in D2D pairs [100].

3) *Mix-Mode D2D Communication*: When several working modes coexist and are optional for D2D pairs, the mode selection becomes a required optimization dimension. A greedy coloring based group partitioning algorithm over conflict graph is proposed to maximize network capacity for both overlay and underlay D2D communications in cellular networks [101]. The bipartite stable matching is combined with the coalition formation game to design a joint mode selection and spectrum access scheme in a D2D and cellular coexisting network where D2D pairs have four specific working modes to select [102]. The maximum-weight bipartite matching is exploited to propose a joint mode selection and user association scheme in a D2D enabled multi-cell network, where each user can associate one BS by cellular mode or its own receiver by D2D mode [103]. A mode selection and resource allocation scheme is designed for energy saving in a D2D and cellular coexisting network with hybrid multiple access techniques, which applies the minimum-cost flow to resource allocation among overlay D2D pairs and uses the interference graph to design a heuristic

resource allocation method for underlay D2D pairs [104]. A joint mode selection and resource allocation scheme is proposed for a D2D-enabled NOMA cellular network, where the interlay mode is developed as a special D2D working mode in NOMA systems and coexists with the underlay mode. This scheme utilizes a maximum-weight clique based BnB approach to obtain the optimal solution [105]. The minimum-cost flow is further applied to mode selection and power control for D2D-enabled NOMA cellular networks to improve network connectivity [106].

*Lessons learned 2*: Graph optimization is suitable for D2D communications in all kinds of working modes to significantly enhance spectrum efficiency. For underlay D2D communications, spectrum reuse is the primary resource management issue addressed by graph optimization methods. The maximum-weight bipartite matching can find the optimal solution for channel reuse between one D2D pair and one cellular user to maximize network capacity, if cellular users are assigned to spectrum channels. If spectrum channels are not assigned, a hypergraph-based BnB algorithm can achieve the optimal channel allocation and reuse solution. Moreover, min-cut based algorithms, matching game theory, and graph coloring are applied to one-to-many spectrum reuse and many-to-many spectrum reuse between cellular users and D2D pairs. For overlay D2D communications, bipartite stable matching, maximum-weight clique, and maximum independent set are efficient for channel allocation among D2D pairs to promote spectrum utilization. For mixed-mode D2D communications, graph coloring, bipartite matching, and minimum-cost flow are effective methods to design joint mode selection and resource allocation algorithms in OMA-based systems. In NOMA-based systems, the maximum-weight clique and minimum-cost flow are applicable. Table III summarizes the reviewed resource management approaches using graph optimization in D2D communications along with references. We can observe from the literature review that resource coordination for social-aware and multi-hop D2D communications remains a worthy issue for future investigation using graph optimization.

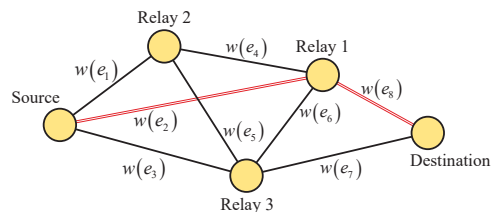
### C. Multi-Hop Networks

Multi-hop networks leverage the cooperation among transmission links to ensure network connectivity, which improves networking flexibility and robustness. The development of multi-hop networks facilitates the emergence of cooperative cellular networks and multi-hop D2D communications. Traditionally, graph optimization methods focus on addressing link scheduling and routing design in multi-hop networks in forms of ad hoc networks, mesh networks, or sensor networks [9], [47], [107], [108]. In recent research literature, graph optimization concentrates on three specific scenarios of multi-hop networks: self-organizing networks (SONs), relay networks, and vehicular networks. Cooperative scheduling among multiple links is primary feature as well as main challenge for resource management in multi-hop networks. In this subsection, we present an overview of the application of graph optimization methods in multi-hop networks over the past decade.

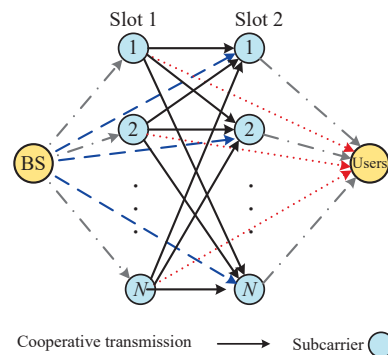
Table III  
A SUMMARY OF RESOURCE MANAGEMENT APPROACHES USING GRAPH OPTIMIZATION IN D2D COMMUNICATIONS

D2D Modes	References	Methods	Graph Types	Issues	RATs
Underlay D2D	[88]	Maximum-weight matching	Bipartite graph	Channel reuse and power allocation	FDMA
	[89]	Heuristic	Conflict graph	Channel reuse	SCMA
	[90]	Greedy coloring	Hypergraph	Channel reuse	NOMA
	[91]	BnB	Hypergraph	Channel allocation and reuse	FDMA
	[92]	Min-cut	Interference graph	Transmission direction	TDD
	[93]	Minimum-weight $k$ -cut	Undirected graph	Spectrum reuse	FDMA
	[94]	Greedy coloring	Hypergraph	Channel allocation	OFDMA
	[95]	Many-to-one matching	Social bipartite graph	Channel allocation and reuse	FDMA
Overlay D2D	[96]	$k$ -coloring	Interference graph	RB assignment and power allocation	OFDMA with full duplex
	[97]	Stable matching	Bipartite graph	Spectrum reuse	FDMA with CR
	[98]	Stable matching	Social-physical graph	Data dissemination	OMA
	[99]	Heuristic	Directed weighted graph	Duplex mode selection	FDMA with full duplex
Mix-Mode D2D	[100]	Maximum-weight clique/Maximum independent set	Conflict graph	Access control, power allocation, and network coding scheduling	Network coding
	[101]	Greedy coloring	Conflict graph	Group partitioning	OFDMA
	[102]	Stable matching with coalition formation game	Bipartite graph	Mode selection and spectrum access	FDMA
	[103]	Maximum-weight matching	Bipartite graph	Mode selection and user association	OMA
	[104]	Minimum-cost flow/Heuristic	Flow network/Interference graph	Mode selection and resource allocation	SCMA and OFDMA
	[105]	Maximum-weight clique	Conflict graph	Mode selection and resource allocation	NOMA
	[106]	Minimum-cost flow	Flow network	Mode selection and resource allocation	NOMA

1) *SONs*: The SON is a representative of multi-hop networks, where the nodes establish wireless connection with each other in a distributed or decentralized manner. Graph optimization is mainly used to design algorithms for link scheduling and resource allocation in SONs. Over the conflict graph, the MWIS is used to formulate the link scheduling problem for SONs with deterministic channel models and then exploited to study the cross-layer optimization in a distributed way [109]. The maximum clique is used to describe and analyze a decentralized link activation strategy in a Rayleigh fading environment by means of random graph theory. In this work, the existence of an edge between any two vertices is set by a probability related to exponential distribution [110]. A  $k$ -coloring based distributed resource allocation algorithm is further proposed to improve the efficiency of resource reuse [111]. Given a topology graph, a greedy link scheduler is designed for SONs with Gaussian multiple access and broadcast channels [112], [113]. For an integrated sensing and communications (ISAC)-aided SON, a shortest path based resource allocation scheme is proposed over a random topology graph to reduce transmission delay, where the weight of each edge follows the exponential distribution [114]. Over the bipartite graph, A many-to-one matching based spectrum allocation scheme is proposed for a SON based on IEEE 802.15.4m to lower spectrum congestion and packet-dropping probability [115]. A maximum matching policy is designed for decentralized medium access control in wireless sensor networks [116], which is further incorporated with double auction game for spectrum allocation to increase the user capacity [117]. In addition, time expanded graph (TEG) is



(a) Shortest path for multi-hop DF relaying networks.



(b) Minimum-cost flow for cooperative cellular networks.

Figure 8. Examples of graph optimization for relay networks.

used to study the cooperative link scheduling in a multi-hop network with multiple channels and multiple slots [118].

2) *Relay Networks*: The relay is a specific infrastructure in wireless networks to interconnect the source node and the destination node by receiving information from the former

and deliver it to the latter. It has numerous advantages on coverage extension, link improvement and energy efficiency. Decode-and-forward (DF) and amplify-and-forward (AF) are two most common relaying strategies. A DF relay decodes, remodulates, and retransmits the received signal, while an AF relay just amplifies and retransmits the received signal without decoding. Graph optimization methods are usually utilized to relay selection and channel assignment for relay networks.

The max-flow min-cut theorem is utilized to devise a directed acyclic graph (DAG) based analytical method, demonstrating how DF relaying substantially enhances energy efficiency in wireless multicasting networks, particularly focusing on a single-source node scenario [119]. An optimal channel and relay assignment scheme is proposed which utilizes maximum-weight matching to allocate each source-destination pair one available relay for the sum-rate maximization in a two-way AF relaying OFDMA network [120]. For a multi-hop relaying network, a shortest path based DF cooperative strategy is proposed to find a path with low bit error rate from the source node to the destination node [121], as shown in Fig. 8a. In Fig. 8a, each intermediate vertex represents a relay node and each edge represents an existing transmission link, whose weight represents the link quality. The bipartite matching is utilized to further design a path selection algorithm for a multi-hop relaying network with multiple source and destination nodes to increase relaying link throughput [122].

Cooperative cellular networks are a cost-effective network architecture to enhance cell coverage and link robustness by deploying relay stations around the BS. To improve network capacity, a maximum-weight clique based spectrum allocation and relay selection scheme is proposed in a relay-assisted bidirectional OFDMA cellular network [123]. Furthermore, the minimum-cost flow based scheme is designed to asymptotically optimize relay selection and resource allocation in a cooperative downlink OFDMA network [124], as per Fig. 8b. In Fig. 8b, each vertex represents a subcarrier in different slots and there are three types of edges, i.e., black solid, blue dotted, and red dotted edges, which correspond to different subcarrier and slot assignments for the relay node. For federated learning in a NOMA relay-assisted IoT network, a greedy MWIS algorithm is employed to efficiently allocate spectrum resources and relay stations to each IoT device, thereby reducing energy consumption during the upload of local model parameters [125].

3) *Vehicular Networks*: Vehicular networks have been one of the most advanced application of IoT, which are currently built on vehicular ad hoc network and vehicle-road cooperation. The mobility of vehicles leads to temporal and spatial changes of network topology, which presents new challenges for resource management [126], [127]. Vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) are two primary categories of communication links in vehicular networks.

For V2V communications, the bipartite matching is widely applied to resource allocation and sharing. A joint secure relay selection and spectrum allocation algorithm is proposed which exploits the maximum matching over a random bipartite graph to assign each V2V pair with one subcarrier to reduce the outage probability [128]. The maximum-weight bipartite

matching is further used to radio resource allocation for vehicle platooning control [129] and spectrum sharing between cellular uplinks and V2V communications [130]. In addition to bipartite matching, the minimum-cost flow is utilized to realize a decentralized link scheduling for data dissemination via V2V links [131]. For V2V and V2I hybrid communications, the interference graph is used to model the network through a similar way for modeling cellular and D2D hybrid networks. Based on constructed interference graph, a heuristic spectrum sharing scheme is proposed between V2V and V2I links [132]. The maximum-weight bipartite matching is also utilized to formulate spectrum sharing problem between V2V and V2I communications for increasing spectrum efficiency [133]. The  $k$ -coloring is applied to channel allocation for computation offloading of V2I and V2V links in edge computing assisted vehicular networks [134].

*Lessons learned 3*: Multi-hop networks are crucial for the application of graph optimization in resource management. In SONs, graph optimization approaches focus on link scheduling and resource allocation. For link scheduling, MWIS, maximum clique, and TEG-based algorithms are exploited to maximize network connectivity. For resource allocation, graph coloring, the shortest path, and bipartite matching are utilized for improving transmission efficiency and resource utilization. In relay networks, relay selection and channel assignment are two main problems addressed by graph optimization methods. Bipartite matching and the shortest path are two popular algorithmic approaches in AF and DF relaying networks. For cooperative cellular networks, minimum-cost flow is used to asymptotically optimize relay selection and resource allocation. Moreover, the maximum-weight clique and MWIS are efficient graph optimization tools in cooperative cellular networks. In vehicular networks, different graph optimization methods are exploited for V2V and V2I communications. For V2V communications, bipartite matching is widely used for resource allocation and sharing. For V2V and V2I communications, interference graph-based algorithm and the maximum-weight bipartite matching are utilized for spectrum sharing to increase spectrum efficiency. Table IV summarizes the reviewed resource management approaches using graph optimization in multi-hop networks along with references. From the literature review, we can see how to deal with resource management problems in future multi-hop networks to meet the requirements of high-mobility, high-reliability, and low-latency applications is a crucial challenge for graph optimization approaches.

#### D. Multi-Antenna Systems

Multi-antenna systems are known as multiple-input-multiple-output (MIMO) systems as well, in which multi-antenna array can smoothly be set up at the transmitter and/or the receiver in diverse wireless networks to increase transmission rate. Graph optimization approaches have been applied to channel allocation and pilot placement in multi-antenna systems.

For channel allocation in multi-antenna systems, the  $k$ -clique is used to formulate the multi-channel sharing in a

Table IV  
A SUMMARY OF RESOURCE MANAGEMENT APPROACHES USING GRAPH OPTIMIZATION IN MULTI-HOP NETWORKS

Networks	References	Methods	Graph Types	Issues	RATs
SONs	[109]	MWIS	Conflict graph	Distributed link scheduling	Deterministic channel model
	[110]	Maximum clique	Random graph	Decentralized link activation	Interference channel access
	[111]	$k$ -coloring	Topology graph	Distributed resource allocation	Distributed access
	[112], [113]	Heuristic	Topology graph	Link scheduling	Gaussian multiple access
	[114]	Shortest path	Topology graph	Resource allocation	ISAC
	[115]	Many-to-one matching	Bipartite graph	Spectrum allocation	CSMA/CA
	[116]	Maximum matching	Bipartite graph	Decentralized medium access control	Slotted random access
	[117]	Maximum matching with double auction game	Bipartite graph	Spectrum allocation	CR
	[118]	Max-flow min-cut	TEG	Cooperative link scheduling	Network coding
Relay Networks	[120]	Maximum-weight matching	Flow network	Channel and relay assignment	AF
	[121]	Shortest path	Directed graph	Path selection and power allocation	Ultra-wideband DF
	[122]	Maximum-weight matching	Bipartite graph	Path selection and power allocation	DF
	[123]	Maximum-weight clique	Conflict graph	Spectrum allocation	Cooperative bidirectional OFDMA
	[124]	Minimum-cost flow	Flow network	Spectrum allocation, relay selection, and transmission mode	Cooperative OFDMA
	[125]	MWIS	Conflict graph	Spectrum and relay allocation	Cooperative NOMA
Vehicular Networks	[128]	Maximum matching	Random bipartite graph	Secure relay selection and spectrum allocation	V2V with DF
	[129]	Maximum-weight matching	Bipartite graph	Radio resource allocation for vehicle platooning control	LTE-V2V
	[130]	Maximum-weight matching	Bipartite graph	Spectrum sharing	Underlay vehicular D2D with OFDMA
	[131]	Minimum-cost flow	Bipartite graph	Decentralized link scheduling	DSRC-V2V
	[132]	Heuristic	Interference graph	Spectrum sharing	V2V and V2I
	[133]	Maximum-weight matching	Interference graph	Spectrum sharing	V2V and V2I
	[134]	$k$ -coloring	Interference graph	Channel allocation	V2V and V2I

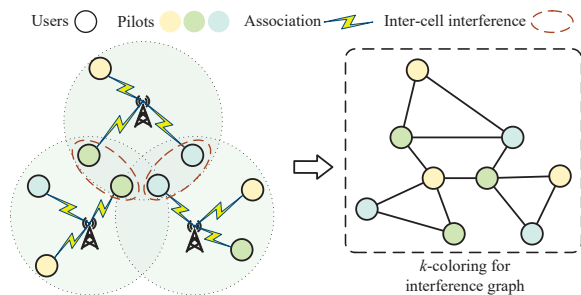


Figure 9. Pilot assignment via  $k$ -coloring in uplink massive MIMO networks.

single-cell multi-user MIMO (MU-MIMO) system, revealing the non-deterministic polynomial-time hardness of this class of problems [135]. To avoid high computational complexity, a  $k$ -coloring based greedy spectrum sharing is proposed to find near-optimal sum-rate of secondary users in a CR MIMO network [136]. Furthermore, many-to-many matching over bipartite graph is exploited to formulate the user-beam association in a massive MIMO system for the sum-rate maximization [137].

Pilot and other training resources are essential for channel estimation in multi-antenna systems. To mitigate pilot contamination due to pilot reuse in multi-cells massive MIMO systems, the  $k$ -coloring is exploited to allocate orthogonal

pilots uplink users in different cells [138], as per Fig. 9. A chromatic number based training resource allocation is proposed to find the minimum number of colors required for multi-cell MIMO systems to decrease the overall training overhead [139]. To resolve pilot collision in a single-cell massive MIMO system, the bipartite graph is used to propose a pilot random access protocol with successive interference cancellation for maximizing the number of active users [140].

*Lessons learned 4:* Channel allocation and pilot placement are two main concerns for graph optimization in multi-antenna systems. For channel allocation, clique-based algorithms, graph coloring, and bipartite matching are utilized to maximize the sum-rate of single-cell and multi-cell MIMO systems. For pilots and other training resources, graph coloring is the most popular approach to improve resource utilization and decrease training overhead. The literature review demonstrates that graph optimization is expected to handle resource management in future massive MIMO systems and multi-antenna systems at mmWave and THz bands.

#### E. Edge Caching and Computing

Computation and storage resources at the edge of various wireless networks have been increasingly important for resource management in line with communication resources. On the one hand, utilizing the storage resource of edge devices to

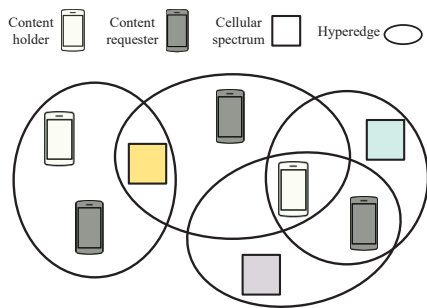


Figure 10. An example of hypergraph model for edge caching.

cache popular contents is an effective approach to overcome backhaul link congestion and reduce content delivery latency. This facilitates the development of edge caching. On the other hand, deploying computing resources close to end users is able to accelerate the execution of compute-intensive tasks from end users via offloading. This prompts the emergence of edge computing. This subsection reviews the research literature on the application of graph optimization to resource management in edge caching and computing.

1) *Edge Caching*: Graph optimization approaches are mainly applied to content placement/delivery scheduling in edge caching. The interference graph is used to model the content delivery in small-cell networks in which each vertex represents one association between a user and a small BS with one channel and an edge connects two vertices with strong interference when delivering requested contents. Based on the constructed interference graph, the maximal independent set is used to propose joint user association and channel assignment algorithm to maximize the system throughput on content delivery [141]. The maximal independent set over interference graph is further employed to optimize user association and BS muting to maximize the number of users simultaneously served by content delivery [142]. In a downlink F-RAN, the MWIS is exploited to design a joint user association and power control scheme for enhanced remote radio heads, i.e., small cells, meanwhile a greedy coloring solution is devised for channel allocation in the central cloud BS, i.e., the macro cell [143]. Moreover, the hypergraph is used to formulate a three-dimensional matching in a cache-enabled D2D underlying cellular network. In this work, there are three types of vertices representing content holders, content requesters, and cellular spectrum resources, respectively. A hyperedge consists of a cellular spectrum resource, a content holder and a content requester, which represents a feasible matching of them [144], as per Fig. 10. The one-to-one stable matching is utilized to spectrum allocation in cache-enabled vehicular networks for maximizing the content delivery efficiency and transmission rate [145].

For content placement, a hypergraph model is proposed to describe the presence of social communities of users and then used to develop a content placement framework in an overly D2D network [146]. A chromatic number based algorithm is proposed for content placement in HetNets for the hit rate maximization, which aims to cache popular contents using smallest memory of small BSs [147]. The minimum-

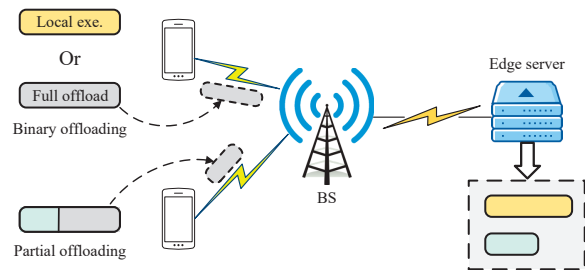


Figure 11. Binary offloading and partial offloading.

weight clique is utilized to a joint user scheduling and content placement scheme in HetNets to optimize the end-to-end throughput, in which a coded multicasting is used to reduce the backhaul traffic load [148]. The maximum-weight perfect matching is used to design a joint content placement and delivery strategy in a cache-enabled NOMA cellular network, aiming at minimizing the average system latency including backhaul-link transmission delay and content delivery delay [149].

2) *Edge Computing*: Graph optimization focuses on two categories of computation resource managements at the edge of wireless networks, i.e., baseband computing scheduling and computation offloading. First, the baseband computing scheduling aims to take full advantage of computation resources at the baseband unit to support users' access requirements. The maximum-weight bipartite matching is utilized to assign each user or its task with available virtual machines (VMs), where baseband computation resources are modeled as different VMs [150], [151]. Second, computation offloading enables users to offload their compute-intensive tasks to nearby BS equipped with edge computing server. Then, BS can execute the compute-intensive task for users. Generally, there are two offloading policies that are the binary offloading and the partial offloading, as per Fig. 11.

For the binary offloading, each user executes the task in local or entirely offload the task to BS. The many-to-one matching is exploited to optimize binary offloading decision and channel assignment in a single-cell mobile edge computing (MEC) network [59]. The MWIS is further used to optimize user clustering and access control for task offloading in single-cell and multi-cell MEC networks [152], [153]. Considering the dependency among computation tasks, the DAG is utilized to describe the execution order and relationship between tasks and propose the corresponding computation offloading scheme [154]. A minimum-cost flow based algorithm is developed to optimize the task offloading in multi-cell MEC networks, which is shown to be applicable to both binary and partial offloading policies [155]. The shortest path is utilized to optimize transmission scheduling in NOMA assisted internet of video things (IoVT) [156].

For the partial offloading, each user can offload a part of its compute-intensive task by partitioning the entire task into subtasks and then execute the remainder in local. The shortest path is exploited to find a proper data routing path for delivering each offloaded subtask and each processing result for maximizing the network throughput in a trackside MEC

network [157]. A bipartite matching based heuristic algorithm is proposed to allocate each subtask one of computation resources in a cloud-edge-end three tier networks for transmit energy minimization [158].

*Lessons learned 5:* With the development of edge caching and computing, storage and computation resources at the edge of wireless networks have become indispensable for resource management, along with communication resources. For edge caching, graph optimization approaches focus on content placement and delivery scheduling. Independent set-based algorithms over interference graphs are efficient for content delivery to maximize system throughput and user capacity. Hypergraph and bipartite matching are also used for content delivery optimization. Furthermore, various graph optimization methods, such as hypergraph, graph coloring, bipartite matching, and clique-based algorithms, are proposed for content placement to make full use of storage resources to promote the hit rate. For edge computing, graph optimization methods are applied to baseband computing scheduling and computation offloading. For baseband computing scheduling, bipartite matching is the efficient and popular approach to assign virtualized computing resources. For computation offloading, bipartite matching, independent set-based algorithms, DAG-based algorithms, and the minimum-cost flow are utilized for binary offloading decision and channel assignment. The shortest path and bipartite matching are usually exploited for designing partial offloading schemes. Table V summarizes the reviewed resource management approaches using graph optimization for edge caching and edge computing along with references. From the literature review, we can see that graph optimization can be utilized for resource management in future edge AI and data center network.

### F. Non-Terrestrial Networks

Tremendous developments of aerospace technologies and the cost reduction of manufacturing and launching facilitate new use cases and applications of NTN and their integration with all kinds of terrestrial wireless networks. This brings new challenges and problems on resource management in NTNs. There is recent literature exploiting graph optimization approaches to tackle relevant issues in satellite networks and aerial networks.

1) *Satellite Networks:* Link scheduling, user association, and handoff are typical issues of resource management in satellite networks [159], [160]. TEG is the most popular graph model for resource management in satellite networks. TEGs can characterize the potential available communication links, i.e., contacts, at different time slots among different nodes in a given satellite constellation. Generally speaking, a TEG consists of  $T$  layers if the scheduling period has  $T$  time slots. Each layer includes all the network nodes, e.g., on-orbit satellites, ground stations, etc., represented by vertices, and contacts in current time slot represented by horizontal edges. There are vertical edges between two adjacent layers representing the carrying of flow forward. Fig. 12 shows an illustration of the TEG for satellite networks. Over the TEG, the maximum-weight bipartite matching is used for contact planning, i.e., link

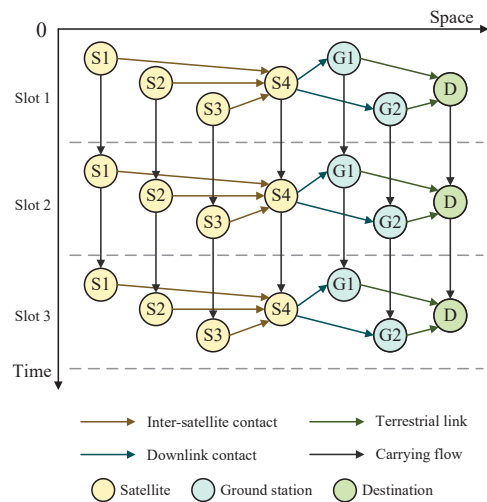


Figure 12. An example of TEG for satellite networks.

scheduling, to maximize the network throughput [161] or the transmission success ratio [162]. The maximum flow is utilized to design a transceiver resource allocation scheme to maximize the resource utilization for inter-satellite communication links [163]. Furthermore, the RL is exploited over the TEG to propose a long-term resource allocation scheme with low computational complexity to maximize the network capacity in a heterogeneous satellite network [164]. The hypergraph is combined with TEGs to model multi-domain resource allocation problems in heterogeneous satellite networks and accomplish the scheduling with low computational complexity [165].

There are several variants of TEG proposed to depict the resource management procedure in satellite networks [166]–[171]. To be specific, a time-evolving resource graph (TERG) is proposed to describe the evolution of multi-dimensional resources in broadband satellite networks [166]. A time-expanded resource relationship graph (TERRG) is further developed to model evolving service capabilities of multi-dimensional resources by a unified measurement standard, which is used to propose an optimal resource mobility utilization strategy [168]. An enhanced TEG (ETEG) is devised which can jointly depicts different resources and combines the transmission and observation phases in satellite networks [170]. Besides the TEG and its variants, the conflict graph is applied to characterize the conflict of resource utilization between different communication links in satellite networks [172], [173].

2) *Aerial Networks:* Due to the characteristics of high maneuverability and low cost, unmanned aerial vehicles (UAVs) have participated in wireless communications and networking to build various aerial networks, as per Fig. 13. In aerial cellular networks, the  $k$ -coloring is employed to assign UAV-BSs with limited channels to maximize downlink sum-rate over a dynamic interference graph, where the edge between any two vertices is dynamically changing due to the mobility of UAV-BSs [174]. The MWIS is utilized to optimize spectrum resource allocation to improve spectrum efficiency of aerial cellular communications [175], [176]. In UAV-assisted

Table V  
A SUMMARY OF RESOURCE MANAGEMENT APPROACHES USING GRAPH OPTIMIZATION FOR EDGE CACHING AND COMPUTING

Use Cases	References	Methods	Graph Types	Issues	Networks
Edge Caching	[141]	Maximal independent set	Interference graph	User association and channel assignment	Small-cell network
	[142]	Maximal independent set	Interference graph	User association and BS muting	Small-cell network
	[143]	MWIS	Interference graph	User association and power control	F-RAN
	[144]	Three-dimensional matching	Hypergraph	D2D pairing and resource allocation	D2D underlying cellular network
	[145]	Stable matching	Bipartite graph	Spectrum allocation	Vehicular content delivery
	[146]	Cooperative game	Hypergraph	Content placement	Overlay D2D
	[147]	Chromatic number	Conflict graph	Content placement	HetNet
	[148]	Minimum-weight clique	Conflict graph	User scheduling and content placement	HetNet with coded multicasting
	[149]	Maximum-weight perfect matching	Undirected graph	Joint content placement and delivery	NOMA
Edge Computing	[150], [151]	Maximum-weight matching	Bipartite graph	VM assignment	C-RAN
	[59]	Many-to-one matching	Bipartite graph	Binary offloading decision	Single-cell MEC
	[152], [153]	MWIS	Conflict graph	User clustering and access control	Multi-cell MEC
	[154]	Extreme value theory	DAG	Dependent task offloading	Time-slotted MEC
	[155]	Minimum-cost flow	Flow network	Binary and partial offloading	Multi-cell MEC
	[156]	Shortest path	Directed graph	Transmission scheduling	IoVT
	[157]	Shortest path	Directed graph	Partial offloading and data routing	Trackside MEC
	[158]	Heuristic	Bipartite graph	Computation resource allocation	Three-tier MEC

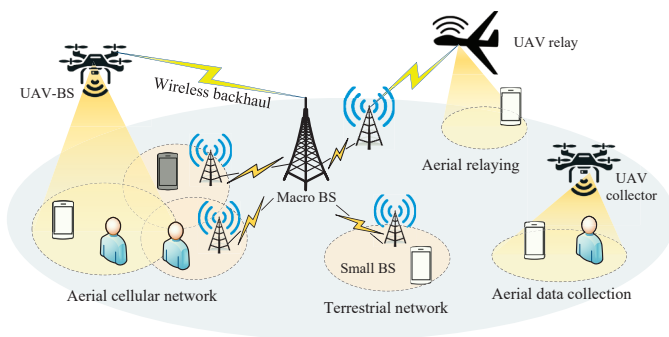


Figure 13. An illustration of aerial networks.

data collection systems, a hypergraph based greedy coloring algorithm is proposed to divide users into several NOMA groups and allocate each group one uplink spectrum channel for the sum-rate maximization [177]. The maximum bipartite matching is exploited for channel allocation to promote energy saving during data collection from sensors to the UAV [178]. For an aerial edge computing network, the bipartite stable matching is used to match users with edge server-mounted UAV-BSs to meet high delay sensitivity requirement for computation offloading [179]. In a UAV-assisted wireless body area networks (WBAN), stable matching and  $k$ -coloring are used to optimize RB allocation to mitigate interference [180].

Besides the above issues, the mobility of UAVs bring new optimization dimension in flexible network deployment. The shortest path is used to obtain the optimal UAV trajectory to minimize the completion time of information uploading from the UAV to ground BSs, where the mobility of UAV follows the fly-hover-fly structure [181]. For a large-scale aerial cellular network, DAG is exploited to describe the trajectory of each UAV-BS and propose a cooperative trajectory planning algorithm over user locations and charging stations [182]. The

maximum flow is utilized to model the trajectory planning problem in a UAV-assisted relay network, which is solved by the spectral graph theory to maximize the data flow of the network [183].

*Lessons learned 6:* Graph optimization has recently been applied to resource management in NTN, including satellite networks and aerial networks, to enhance network capacity and the coverage performance. In satellite networks, the TEG is the most common graph model for resource management. Over TEGs, bipartite matching, the maximum flow, hypergraph, and RL-based algorithms are utilized for contact planning and resource allocation with different objectives. Furthermore, many TEG variants have been proposed to describe the process of resource management and model the evolution of multi-dimensional resources. In aerial networks, graph coloring, independent set-based algorithms, hypergraph, and bipartite matching are used for channel allocation and the association between UAVs and other network elements, e.g., terrestrial BSs and users, to promote resource utilization. More importantly, graph optimization methods including the shortest path, DAG-based algorithms, and the maximum flow are utilized for solving and modeling UAV trajectory planning problems to improve the coverage performance in diverse aerial networks. Table VI summarizes the reviewed resource management approaches using graph optimization in NTN along with references. The literature review demonstrates that graph optimization is promising to manage multi-dimensional resources in future multi-tier space-air-ground integrated networks.

### G. Summary and Discussion

This section investigates the application of graph optimization for resource management in wireless networks. We review the literature from five scenarios: cellular networks, D2D



Table VI  
A SUMMARY OF RESOURCE MANAGEMENT APPROACHES USING GRAPH OPTIMIZATION IN NTN

Networks	References	Methods	Graph Types	Issues	Use Cases
Satellite Networks	[161], [162]	Maximum-weight matching	Bipartite graph	Contact planning	Satellite relaying
	[163]	Maximum flow	Flow network	Transceiver resource allocation	Inter-satellite communications
	[164]	RL	TEG	Long-term resource allocation	Inter-satellite communications
	[165]	Shortest path	Hypergraph	Multi-domain resource allocation	Inter-satellite communications
	[166]	Maximum flow	TERG	Multi-dimensional resource scheduling	Satellite relaying
	[167]	Maximum flow	Event-driven TEG	Multi-resource coordinate scheduling	Earth observation
	[168]	Maximum flow	TERRG	Resource mobility utilization	Earth observation
	[169]	Maximum flow	TEG	Contact planning	Remote sensing
	[170]	Maximum flow	ETEG	Multi-resource coordinate scheduling	Earth observation
	[171]	Maximum flow	Resource TEG	Energy-efficient resource scheduling	Remote sensing
	[172]	Q-learning	Conflict graph	Data forward and backward induction	Remote sensing by small satellites
	[173]	Maximum independent set	Conflict graph	Task scheduling	Satellite relaying
Aerial Networks	[174]	$k$ -coloring	Conflict graph	Channel allocation	Aerial cellular communications
	[175], [176]	MWIS	Conflict graph	Spectrum allocation	Aerial cellular communications
	[177]	Greedy coloring	Hypergraph	User grouping and channel allocation	Data collection
	[178]	Maximum matching	Bipartite graph	Channel allocation	Data collection
	[179]	Stable matching	Bipartite graph	User association	Aerial edge computing
	[180]	Stable matching and $k$ -coloring	Topology graph	RB allocation	WBAN
	[181]	Shortest path	Topology graph	UAV trajectory planning	Data uploading with NOMA
	[182]	Dynamic programming	DAG	Cooperative trajectory planning	Aerial cellular communications
	[183]	Maximum flow	Flow network	UAV trajectory planning	UAV relaying

communications, multi-hop networks, multi-antenna systems, edge computing and caching, and NTN. We first focus on the graph optimization approaches for resource management in cellular networks. Various graph optimization methods are applied in single-cell networks and multi-cell networks to coordinate interference and improve resource utilization. Second, we elaborate on graph optimization methods in D2D communications with different working modes for spectrum reuse and mode selection to increase spectrum efficiency. Then, we investigate graph optimization-based link scheduling and resource allocation algorithms in multi-hop networks, including SONs, relay networks, and vehicular networks. Afterwards, we concentrate on graph optimization methods in multi-antenna systems, which are applied to channel allocation and pilot placement to increase resource utilization and reduce training overhead. Furthermore, we investigate how graph optimization facilitates the development of computation and storage resource management at the edge of wireless networks from the perspectives of edge caching and edge computing. Finally, we review the literature of graph optimization applied in NTN, including satellite networks and aerial networks. TEG-based algorithms are highly effective for contact planning and resource allocation in satellite networks, while diverse graph optimization approaches are used for UAV trajectory

planning and resource allocation in aerial networks. In the future, advanced graph optimization methods will continue to leverage their advantages in combinatorial optimization and combine with emerging wireless communication technologies to enhance overall performance in resource management.

#### IV. CONCLUSION

In this part, we have presented a comprehensive survey on resource management via graph optimization. First, we have started with the basics of graph theory to introduce graph optimization problems and methods. Then, the literature on graph optimization approaches for resource management has been systematically reviewed according to different scenarios, i.e., cellular networks, D2D communications, multi-hop networks, multi-antenna systems, edge caching and computing, and NTN. In Part II of this survey, we will focus on graph learning for resource management in wireless networks and then discuss current technical challenges and future research directions in this field.

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