Plug-and-Play Distributed Estimation of Driving States in an Open Vehicle Platoon

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***Abstract*—The information regarding the driving states of all vehicles is crucial for achieving optimal group performance in a vehicle platoon. This article focuses on the fully distributed driving state estimation problem in open vehicle platoons, which frequently experience arrivals and departures of vehicles. To address this problem, we propose a distributed driving state observer inspired by the leader–follower consensus technique. This observer can reconstruct the global driving state of the platoon, including the positions, velocities, and accelerations of all vehicles. We also derive the necessary and sufficient conditions to ensure the stability of its estimation error dynamics. The proposed observer is highly flexible in platoons with a strongly connected communication network, as it can be constructed and operated using the local knowledge of each vehicle only, without relying on global information of a platoon such as the number of vehicles. We demonstrate the observer's plug-and-play operations in the face of platoon merging and splitting and analyze its estimation stability. Extensive simulation results demonstrate the effectiveness of our theoretical results and the potential of the proposed observer for platoon control.**

***Index Terms*—Distributed observer, flexibility, plug-and-play, vehicle platoon.**

Manuscript received 8 August 2023; accepted 20 October 2023. Date of publication 29 November 2023; date of current version 4 April 2024. This work was supported in part by the National Natural Science Foundation of China under Grant 62293510/62293511, Grant 62273305, and Grant 62003303, in part by Shenzhen Science and Technology Program under Grant JCYJ20210324120011032, in part by the Young Elite Scientist Sponsorship Program by cast of China Association for Science and Technology under Grant YESS20210158. Paper no. TII-23-3021. *(Corresponding author: Chengcheng Zhao.)*

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Color versions of one or more figures in this article are available at https://doi.org/10.1109/TII.2023.3331535.

Digital Object Identifier 10.1109/TII.2023.3331535

I. Introduction

VEHICLE platooning is a cooperative driving mode for a fleet of vehicles, in which vehicles drive close to each other by automation and Vehicle-to-Vehicle (V2V) communication technologies [1]. It has been recognized as a promising technology in intelligent transportation systems (ITS) thanks to its potential in saving energy, reducing exhaust emissions, increasing road capacity, and improving the safety and comfort of drivers [2]. Vehicle platooning has attracted considerable attention, with research focusing on transport planning optimization [3], platoon control [4], and the coupled relationship between traffic dynamics and platoon behaviors [5].

A vehicle platoon is said to be "open" if it experiences joining or leaving of vehicles, i.e., the merge or split of the platoon. The merge and split of the platoon are common in practice due to the varying transport destinations of vehicles and the real-time changing road conditions [6]. There exist many studies on open platoons, mostly focusing on the longitudinal control [7], [8], lateral control [9], [10], and communication protocol design [11]. Considering the crucial role of vehicle driving state in platoon control, this article focuses on the global driving state estimation problem in open platoons. The objective of global driving state estimation is to design an estimation algorithm for each vehicle to determine the positions, velocities, and accelerations of all vehicles in the platoon using only locally available information. The acquisition of the global driving state is of paramount importance for ensuring public safety in platoons. It enables the identification and isolation of malicious vehicles that might broadcast false information and helps mitigate the impact of attacks on vehicles, as highlighted in [12]. Moreover, obtaining the global driving state proves advantageous for each vehicle in terms of improving platoon control performance and optimizing platoon operations. Research has shown that by utilizing driving state information from more preceding or following vehicles in each vehicle's local controller, it is possible to reduce intervehicle distances and achieve faster consensus on acceleration and velocity [13], [14], [15]. Furthermore, in emergency scenarios such as sudden braking or lane changing, access to the global driving state information of the platoon enables each vehicle to respond more effectively. However, due to limitations in the range of onboard sensing and communication devices, each vehicle in a platoon can only access the driving state of vehicles within its neighboring region through sensing and communication technology. Consequently, achieving global driving state estimation for each vehicle using only local information (i.e., information obtained from local sensors and local communication) is a nontrivial challenge. This work is motivated to address these critical and challenging open questions. It aims to develop methods that allow each vehicle to estimate the global driving state of the platoon solely based on locally available information, and to explore the applicability of these methods to open vehicle platoons.

Much attention has been paid to the global driving state estimation problem for vehicle platoons by a centralized fashion [12], [16], [17], [18]. Dutta et al. [12] proposed a resilient estimator for each vehicle to estimate the global driving state in the presence of sensor attacks. Suzuki [16] used two estimators, i.e., a particle filter and an unscented Kalman filter, to estimate the headway and velocities of all vehicles in a six-vehicle platoon with an appropriate human–machine interface (HMI). Wakasa and Sawada [18] designed a distributed partial-state observer, consisting of a local observer running on the traffic management system installed on the road and a group of local observers running on each vehicle, for the position estimation of all vehicles in the platoon. It should be noticed that an extra system acting as a fusion center, such as HMI in [16] and the traffic management system in [18], is required and places additional demands on the construction of transportation infrastructure. Moreover, these estimators were all designed in a centralized way, which means that the update and the parameter choice of these estimators need to use the global information of the platoon, including the number of vehicles [16], [17] and all vehicles' measurements [18]. From the viewpoint of practical application, in a platoon without an extra system and recalling the "openness" of a platoon, the driving state estimation problem for platoons has to be solved in a fully distributed fashion.

As a platoon is a typical networked control system (NCS), there has been a recent surge of interest in distributed driving state estimation [19], [20], [21], [22], [23], [24]. Pan [19] designed a robust nonlinear observer for each vehicle to estimate its own state in the general bicycle model considering uncertainties and external disturbances. Teo et al. [20] proposed an estimation scheme based on the dead reckoning process for each vehicle to estimate the leading vehicle's velocity during communication dropouts. Wen and Guo [21] utilized stochastic analysis techniques to design a reduced-order observer for each vehicle to estimate the relative acceleration of neighboring vehicles in a platoon. Liu et al. [22] developed an extended-state observer for each following vehicle to estimate the relative acceleration between itself and the preceding one only based on the local measurement. Ju et al. [23] designed a distributed Kalman filter (DKF) for each vehicle to estimate its own driving state by fusing the local measurement with the neighbors' measurements. He et al. [24] developed a resilient observer for each vehicle to estimate its own driving state accurately under sensor attacks based on robust techniques. However, these algorithms can only guarantee that each vehicle obtains an estimate of the state of some vehicles in a platoon, including itself, the leading vehicle, and neighboring vehicles. In summary, for an open platoon without any extra system, it remains open to design an estimator for each vehicle to harvest the global driving state of the platoon in a fully distributed fashion.

This article investigates the fully distributed global driving state estimation problem for open platoons. Here, fully distributed design implies that a plug-and-play operation is enabled, allowing each vehicle to automatically reconfigure the estimation algorithm when facing vehicles joining or leaving. We propose a plug-and-play distributed driving state observer that overcomes the limitations of the aforementioned estimators in the context of vehicle platooning. The main contributions of this article are threefold.

1) We propose a distributed driving state observer for each vehicle to estimate the positions, velocities, and accelerations of all vehicles in the platoon. We also derive necessary and sufficient conditions that ensure the stability of the estimation error dynamics of the proposed observer.
2) To ensure the openness of the platoon, we analyze the flexibility of the proposed scheme and provide two plug-and-play schemes to support platoon merging and splitting scenarios.
3) We present an distributed observer-based controller to ensure the internal stability of a vehicle platoon under a constant time headway policy. In addition, through simulations, we demonstrate that the proposed distributed driving state observer is effective and improves platoon control performance.

The rest of this article is organized as follows. In Section II, we present the problem of our interests. The distributed state observer and its operations in open platoons are, respectively, provided in Sections III and IV. Section V verifies the main results through numerical simulations. Finally, Section VI concludes this article.

Notations: For a square matrix $M \in \mathbb{R}^{n\times n}$, we use the symbols $\mathrm{sp}(M) = \{\lambda \in \mathbb{C} | \det(\lambda I - M) = 0\}$ and $\rho(M)$ to denote the set of all eigenvalues and the spectral radius of M, respectively. The symbol $\otimes$ denotes the Kronecker product. Let I_m denote a m-dimension identity matrix. The symbol $\mathbf{1}_m$ is an $m \times 1$ column vector whose elements are all one. Let e_i denote the ith unit vector in $\mathbb{R}^m$. We use $\mathbf{0}$ to denote the all-zero matrix with the appropriate dimension. For two sets $\mathcal{A}$ and $\mathcal{B}$, $\mathcal{A}/\mathcal{B} = \{x | x \in \mathcal{A} \text{ and } x \notin \mathcal{B}\}$.

II. Problem Formulation

A. Communication Network Model

Communication networks play a vital role in determining the information flow among vehicles and are essential for cooperative behavior within a vehicle platoon. Consider a homogeneous platoon with m vehicles, where all vehicles have the same dynamics. Each vehicle in the platoon has a unique ID, denoted by $i = 1, \ldots, m$, where $i = 1$ represents the ID of the leading vehicle. The communication network is modeled as a directed graph $\mathrm{G} = (\mathrm{V}, \mathrm{E})$, where the node set $\mathrm{V} = \{1, \ldots, m\}$ and the edge set $\mathrm{E} \subseteq \mathrm{V} \times \mathrm{V}$ describe the information flow among all vehicles in the platoon. An edge from vehicle i to j, denoted by $(i, j) \in \mathrm{E}$, implies that vehicle i can transmit its information to vehicle j. Note that self-loops are not allowed, i.e., $(i, i) \notin \mathrm{E}$ for all $i \in \mathrm{V}$. We use the symbols $\mathrm{N}_i = \{j \in \mathrm{V} | (j, i) \in \mathrm{E}\}$

and $\mathrm{S}_i = \{j \in \mathrm{V} | (i,j) \in \mathrm{E}\}$ to, respectively, represent the in-neighbor set and out-neighbor set of vehicle i. In other words, information from vehicles in N_i can be received by vehicle i, and information from vehicle i can be received by vehicles in S_i. Finally, we define $d_i = |\mathrm{N}_i|$ as the number of in-neighbors of node i.

In graph G, a directed path of length $l+1$ from node i to node j is an ordered sequence $(i, i_1, \ldots, i_l, j)$ such that every consecutive pair of nodes in the sequence is connected by an edge. In other words, $(i, i_1) \in \mathrm{E}$, $(i_l, j) \in \mathrm{E}$, and $(i_k, i_{k+1}) \in \mathrm{E}$ for all $k \in \{1, 2, \ldots, l-1\}$, where l is less than the total number of nodes m. A directed graph $\mathrm{G}_s = (\mathrm{V}_s, \mathrm{E}_s)$ is a subgraph of G if its set of nodes V_s is a subset of the nodes in G, and its set of edges E_s is a subset of the edges in G. Let $W_{\mathrm{G}} = [w_{ij}] \in \mathbb{R}^{m \times m}$ denote the weighted adjacency matrix of graph G, where $w_{ij} > 0$ if and only if $(i,j) \in \mathrm{E}$, and $w_{ij} = 0$ otherwise. Let $L_{\mathrm{G}} = [l_{ij}] \in \mathbb{R}^{m \times m}$ be the Laplacian matrix of graph G, where l_{ii} is the sum of weights of all edges connected to node i, and l_{ij} is equal to the negative weight of the edge connecting nodes i and j if $i \neq j$, and 0 otherwise. Throughout the rest of this article, the terms "nodes" and "vehicles" are used interchangeably to refer to the objects represented by the nodes in the graph. An assumption and some necessary concepts related to graphs are provided as follows.

Assumption 1: The communication network G is strongly connected, i.e., there is at least one directed path between any two distinct nodes $i, j \in \mathrm{V}$.

Here, a strongly connected communication network of vehicle platoons means that there exists a directed communication path for any pair of vehicles in the platoon. Several platoons have communication networks that satisfy Assumption 1, as exemplified by the k-nearest neighbor platoon discussed in [25]. In this configuration, each vehicle can communicate with k vehicles both in front and behind, maintaining the required connectivity for the platoon.

Definition 1: (see[26], Definition 4.10) A non-negative matrix $M \in \mathbb{R}^{n \times n}$ is row-substochastic if its row-sums are at most 1 and at least one row-sum is strictly less than 1, that is, $M\mathbf{1}_n \leq \mathbf{1}_n$, and there exists $i \in \mathrm{V}$ such that $e_i^\top M \mathbf{1}_n < 1$.

Lemma 1: (see[26], Th. 2.7 and Corollary 4.11) Let M be a matrix that is row-substochastic and G be the directed graph associated with M. M is said to be Schur stable if and only if G contains directed paths from each node with out-degree 1 to a node with out-degree less than 1. Here, the out-degree of a node is defined as the sum of the weights of all its out-edges. Moreover, if M is irreducible, then M is necessarily Schur stable, which is equivalent to saying that the spectral radius $\rho(M)$ is strictly less than 1.

B. Vehicle Dynamics Model

We adopt the linearized longitudinal dynamics of vehicle i in [27] described as follows:

$$\dot{s}_i(t) = v_i(t)$$

$$\dot{v}_i(t) = a_i(t)$$

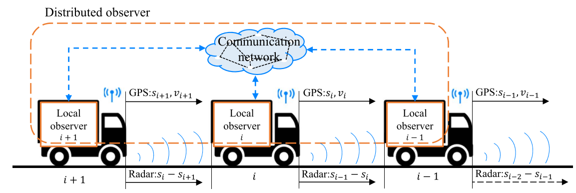


Fig. 1. Distributed driving state observer in a vehicle platoon.

$$\dot{a}_i(t) = -\frac{1}{\tau} a_i(t) + \frac{1}{\tau} u_i(t) \tag{1}$$

where $s_i(t)$, $v_i(t)$, $a_i(t)$, $u_i(t)$, respectively, denote the position, velocity, acceleration, and control input of vehicle i at time t, and τ is an engine time constant representing the inertial lag of vehicle longitudinal dynamics and is identical for all vehicles, i.e., a homogeneous platoon is considered.

In practical terms, the driving state of a vehicle is continuous and analog, but it is sampled by sensors at regular intervals. The control systems are implemented digitally in on-board computers, operating at discrete time instants. Therefore, we discretize the above dynamics using a sampling period of τ_s, resulting in the following third-order state-space model of vehicle i in a homogeneous platoon

$$x_i(k+1) = Ax_i(k) + Bu_i(k), \forall i \in \mathrm{V} \tag{2}$$

where $x_i(k) = [s_i(k), v_i(k), a_i(k)]^\top$ is the driving state of vehicle i, and

$$A = \begin{bmatrix} 1 & \tau_s & \frac{\tau_s^2}{2} \\ 0 & 1 & \tau_s \\ 0 & 0 & 1 - \frac{\tau_s}{\tau} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{\tau_s}{\tau} \end{bmatrix}.$$

Fig. 1 illustrates our problem, in which the leading vehicle can obtain its real-time position and velocity from a GPS mounted on the vehicle, while the following vehicles in the platoon can obtain their own positions and velocities through GPS and intervehicle distance measurements from radar. As a result, the sensor measurements for vehicle i can be expressed as follows:

$$y_i(k) = \begin{cases} C_{1,1} x_1(k), \ i = 1 \\ C_{i,i} x_i(k) + C_{i,i-1} x_{i-1}(k), \ i \in \mathrm{V}/\{1\} \end{cases} \tag{3}$$

where

$$C_{1,1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, C_{i,i} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, C_{i,i-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Stacking all vehicles' states into a single vector and the measurements of all vehicles together into a single vector, we obtain the system dynamics in a compact form as follows:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k)$$

$$y_i(k) = \begin{cases} C_1 \mathbf{x}(k), \ i = 1 \\ C_i \mathbf{x}(k), \ i \in \mathrm{V}/\{1\} \end{cases} \tag{4}$$

where $\mathbf{x}(k) = [x_1(k)^\top, \ldots, x_m(k)^\top]^\top \in \mathbb{R}^{3m}$, $\mathbf{A} = I_m \otimes A$, $\mathbf{u}(k) = [u_1(k)^\top, \ldots, u_m(k)^\top]^\top \in \mathbb{R}^{3m}$, $\mathbf{B} = I_m \otimes B$, $C_1 =$

$e_1^\top \otimes C_{1,1} \in \mathbb{R}^{3\times 3m}$ for the leading vehicle and $C_i = [0,\ldots,0,C_{i,i-1},C_{i,i},0,\ldots,0] \in \mathbb{R}^{3\times 3m}$ for the following vehicles. Observability analysis is necessary for the design of estimation algorithm. Based on (2)–(4), we characterize the observability of the platooning system at the vehicle level and the platoon level, respectively, as follows.

1) The pair $(A, C_{i,i}), \forall i \in \mathrm{V}$ is observable, by which each vehicle can infer its own driving state from local measurements.
2) The pair $(\mathbf{A}, C_i), \forall i \in \mathrm{V}$ is unobservable. That is, the driving state of the platoon cannot be fully estimated from a single vehicle directly.
3) The pair $(\mathbf{A}, \mathbf{C})$ is observable with $\mathbf{C} = [C_1^\top,\ldots,C_m^\top]^\top$. It implies that the platooning system (4) is jointly observable, i.e., the global state can be estimated by combining all vehicles' measurements.

C. Problem of Interests

To ensure that an open vehicle platoon operates in a safer and more optimized way, we focus on the global driving state estimation problem. Specifically, we aim to propose a fully distributed design of distributed driving state observer, which consists of a group of local observers deployed on each vehicle, and answer the following questions.

Q1. Are there any gain parameters available that guarantee the proposed observer's output asymptotically converges to the actual driving state of the platoon?

Q2. How can we ensure the observer's adaptability for a plug-and-play operation? How does the observer work in situations involving platoon merging and splitting?

Q3. How can we use the output of the proposed observer to improve platoon control performance?

III. Distributed Driving State Estimation Scheme

In this section, we develop a distributed driving state observer for a vehicle platoon and establish the conditions that guarantee the stability of its estimation error dynamics.

A. Distributed Observer Design

We begin by presenting an overview of our design. Based on the observability analysis of the platooning system, each vehicle $i \in \mathrm{V}$ is able to estimate its own state $x_i(k)$ using a Luenberger observer and its local measurement $y_i(k)$. The estimate is denoted as $\bar{x}_i(k)$ and is updated by vehicle i locally. The estimate $\bar{x}_i(k)$ is then treated as an external signal to be tracked by all vehicles in the platoon using a leader–follower consensus technique. This allows each vehicle i to construct an estimate of vehicle j's state by tracking the signal $\bar{x}_j(k)$ generated by vehicle j using its own local Luenberger observer for $\forall j \in \mathrm{V}$. We represent the estimate of vehicle j as $\hat{x}_i^{(j)}(k)$ by vehicle $i \in \mathrm{V}$. Finally, each vehicle i develops an estimate of the true global driving state $\mathbf{x}(k)$, denoted as $\hat{\mathbf{x}}_i(k) = [(\hat{x}_i^{(1)}(k))^\top,\ldots,(\hat{x}_i^{(m)}(k))^\top]^\top$. This distributed state estimation procedure can be summarized as follows:

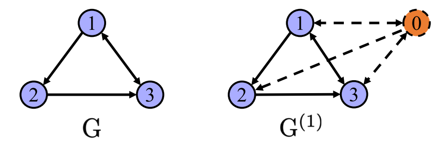


Fig. 2. Example of the graph G and $\mathrm{G}^{(j)}$ $(j = 1)$.

$$\bar{x}_i(k+1) = \begin{cases} A\bar{x}_1(k) + Bu_1(k) + F_1(y_1(k) - C_{1,1}\bar{x}_1(k)),\ i = 1 \\ A\bar{x}_i(k) + Bu_i(k) + F_i(y_i(k) - (C_{i,i}\bar{x}_i(k) \\ +C_{i,i-1}\hat{x}_i^{(i-1)}(k))),\ i \in \mathrm{V}/\{1\} \end{cases} \tag{5a}$$

$$\hat{x}_i^{(j)}(k+1) = A(\hat{x}_i^{(j)}(k) + \sum_{l\in \mathrm{N}_i} w_{il}^{(j)}(\hat{x}_l^{(j)}(k) - \hat{x}_i^{(j)}(k)) + w_{i0}^{(j)}(\bar{x}_j(k) - \hat{x}_i^{(j)}(k))) + Bu_j(k), i, j \in \mathrm{V} \tag{5b}$$

where F_1 and F_i are the Luenberger observer gains, $w_{il}^{(j)}$ and $w_{i0}^{(j)}$ are nonnegative consensus weights related to the communication network G. These parameters will be determined by the following stability analysis of the estimation error dynamics of the proposed observer (5). Note that the information transmitted by each vehicle $i \in \mathrm{V}$ through the communication network G is its estimate of the global driving state of the platoon, i.e., $\hat{\mathbf{x}}_i(k)$.

B. Stability Analysis of Estimation Error Dynamics

In this part, we explore the conditions that guarantee the stability of the estimation error dynamics of the proposed observer, as given in (5).

To accomplish this, we define a set of virtual graphs for each vehicle j, denoted by $\{\mathrm{G}^{(j)} = (\mathrm{V}^{(j)}, \mathrm{E}^{(j)})\}_{j\in \mathrm{V}}$. The set $\mathrm{V}^{(j)} = \{0, 1,\ldots,m\}$ consists of a virtual node representing a copy of vehicle j (denoted by 0), and the remaining numbers $1,\ldots,m$ are used to index the other vehicles in the graph G. The virtual node 0 has the same neighbors as node j and acts as an "exosystem" for the actual vehicles to follow its dynamical state. The set $\mathrm{E}^{(j)} \subseteq \mathrm{V}^{(j)} \times \mathrm{V}^{(j)}$ is composed of three parts: 1) All edges of graph G; 2) A bidirectional edge between node 0 and j; 3) The edges between node 0 and node j's neighbors, including both in-neighbors and out-neighbors, which are identical to the pattern of connections between node j and its neighbor nodes. To illustrate this concept, a three-node example is given in Fig. 2, where the solid lines represent the actual communication links, and the dashed lines represent the virtual links constructed for the estimation scheme design. The figure shows graph G and virtual graph $\mathrm{G}^{(j)}$ obtained by virtualizing node j as an exosystem.

Following the terminology used in graph G, we use $\mathrm{N}_i^{\mathrm{G}^{(j)}} = \{j \in \mathrm{V}^{(j)} | (j, i) \in \mathrm{E}^{(j)}\}$ and $\mathrm{S}_i^{\mathrm{G}^{(j)}} = \{j \in \mathrm{V}^{(j)} | (i, j) \in \mathrm{E}^{(j)}\}$ to denote the set of node i's in-neighbors and the set of out-neighbors in graph $\mathrm{G}^{(j)}$, respectively. Note that $\mathrm{G} = (\mathrm{V}, \mathrm{E})$ is a subgraph of $\mathrm{G}^{(j)}$ as G can be attained from $\mathrm{G}^{(j)}$ by removing node 0 from $\mathrm{V}^{(j)}$ and removing the edges rooted at node 0 from $\mathrm{E}^{(j)}$. We use $W_{\mathrm{G}^{(j)}} = [w_{ij}^{(j)}] \in \mathbb{R}^{(m+1)\times(m+1)}$

and $L_{\mathrm{G}^{(j)}} = [l_{ij}^{(j)}] \in \mathbb{R}^{(m+1)\times(m+1)}$, respectively, to represent a weighted adjacency matrix and a Laplacian matrix of $\mathrm{G}^{(j)}$, where for $i \in \mathrm{V}^{(j)}$, $w_{ij}^{(j)} > 0$ if $(i,j) \in \mathrm{E}^{(j)}$ and $w_{ij}^{(j)} = 0$ otherwise, and $l_{ii} = \sum_{j=0}^{m} w_{ij}^{(j)}$ and $l_{ij} = -w_{ij}^{(j)}$ if $i \neq j$. We define $P_{\mathrm{G}^{(j)}} = I_{m+1} - L_{\mathrm{G}^{(j)}} = [p_{ij}^{(j)}] \in \mathbb{R}^{(m+1)\times(m+1)}$, which is called Perron matrix of $\mathrm{G}^{(j)}$ [28]. The matrix $P_{\mathrm{G}^{(j)}}$ can be partitioned into four blocks as follows:

$$P_{\mathrm{G}^{(j)}} = \left[\begin{array}{c|c} 1 - \sum_{i=1}^{m} w_{0i}^{(j)} & \left[w_{01}^{(j)}, \ldots, w_{0m}^{(j)}\right] \\ \hline O^{(j)}\mathbf{1}_m & Q^{(j)} \end{array}\right] \tag{6}$$

where $O^{(j)}$ is an m-by-m diagonal matrix in which the ith diagonal element is $w_{i0}^{(j)}, i \in \mathrm{V}$. A fact about $O^{(j)}$ is that $O^{(j)}\mathbf{1}_m = \mathbf{1}_m - Q^{(j)}\mathbf{1}_m = (I_m - Q^{(j)})\mathbf{1}_m$ since $P_{\mathrm{G}^{(j)}}\mathbf{1}_{m+1} = \mathbf{1}_{m+1}$. The condition to ensure that the matrix $Q^{(j)}$ is Schur stable is given in the following lemma.

Lemma 2: The submatrix $Q^{(j)} \in \mathbb{R}^{m\times m}, \forall j \in \mathrm{V}$ of $P_{\mathrm{G}^{(j)}}$ is Schur stable, i.e., all the eigenvalues of $Q^{(j)}$ have modulus smaller than 1, iff G is strongly connected.

Proof. Sufficiency: The associated graph of $Q^{(j)}$ is G. Thus, $Q^{(j)}$ is irreducible due to the hypothesis that G is strongly connected. Meanwhile, $Q^{(j)}$ is row-substochastic by Definition 1. By Lemma 1, $Q^{(j)}$ is Schur stable.

Necessity: Suppose that G is not strongly connected. Then we can always find at least two nodes, termed as node i to node j, so that there is no path from node i to node j. We look at graph $\mathrm{G}^{(j)}$ and consider the associated matrix $Q^{(j)}$. Note that there is no path from node i to node 0 as there is no path from node i to j and that a bidirectional edge exists between node 0 and j as the former is a virtual copy of the latter. Therefore, we focus on the subgraph restricted to G in $\mathrm{G}^{(j)}$ the out-degree of node i is 1 and out-degree of node j strictly less than 1. By Lemma 1, the matrix $Q^{(j)}$ is not Schur stable, which completes the proof. ■

On this basis, we present the main result about the stability of the estimation error dynamics of the distributed observer (5) when the control input $\mathbf{u}(k)$ is known to all vehicles. The details are as follows.

Theorem 1: Consider a vehicle platoon system (4) over a communication network G satisfying Assumption 1. Under distributed observer (5), the estimate $\hat{\mathbf{x}}_i(k)$ for all $i \in \mathrm{V}$ asymptotically converges to $\mathbf{x}(k)$, i.e., $\lim_{k\to\infty} \|\hat{\mathbf{x}}_i(k) - \mathbf{x}(k)\| = 0$, iff the gain F_i is chosen such that $\rho(A - F_i C_{i,i}) < 1$, and the weights $w_{i0}^{(j)} \geq 0$ and $w_{il}^{(j)} \geq 0$ satisfy $\sum_{l \in \mathrm{N}_i^{\mathrm{G}^{(j)}}} w_{il}^{(j)} \leq 1$, for all $j \in \mathrm{V}$.

Proof: We define the estimation error as

$$\begin{aligned} e_0^{(j)}(k) &= \bar{e}_j(k) \triangleq \bar{x}_j(k) - x_j(k) \\ e_i^{(j)}(k) &\triangleq \hat{x}_i^{(j)}(k) - x_j(k) \end{aligned} \tag{7}$$

for $i,j \in \mathrm{V}$. Recall that the state estimate to vehicle j by the virtual node 0 in $\mathrm{G}^{(j)}$ is $\bar{x}_j$. Substituting (2), (3), (5) into (7), we have

$$e_0^{(j)}(k+1) = \bar{e}_j(k+1)$$
$$= \begin{cases} (A - F_1 C_{1,1})\bar{e}_1(k), \; j = 1 \\ (A - F_j C_{j,j})\bar{e}_j(k) - F_j C_{j,j-1} e_j^{(j-1)}(k), \\ \hfill j \in \mathrm{V}/\{1\} \end{cases} \tag{8a}$$

$$\begin{aligned} e_i^{(j)}(k+1) = A\Big(e_i^{(j)}(k) &+ \sum_{l \in \mathrm{N}_i} w_{il}^{(j)}\big(e_l^{(j)}(k) - e_i^{(j)}(k)\big) \\ &+ w_{i0}^{(j)}\big(e_j^{(j)}(k) - e_i^{(j)}(k)\big)\Big), \; i,j \in \mathrm{V}. \end{aligned} \tag{8b}$$

Let $e^{(j)}(k) = [e_0^{(j)}(k)^\top, e_1^{(j)}(k)^\top, \ldots, e_m^{(j)}(k)^\top]^\top$. Based on the error dynamics (8), we obtain the following composite error dynamics:

$$\begin{aligned} e^{(j)}(k+1) = &\underbrace{\left[\begin{array}{c|c} A - F_j C_{j,j} & \mathbf{0} \\ \hline O^{(j)} \otimes A & Q^{(j)} \otimes A \end{array}\right]}_{R^{(j)}} e^{(j)}(k) \\ &+ \underbrace{\left[\begin{array}{c|c} \mathbf{0} & e_{j-1}^\top \otimes (-F_j C_{j,j-1}) \\ \hline \mathbf{0} & \mathbf{0} \end{array}\right]}_{S^{(j)}} e^{(j-1)}(k). \end{aligned} \tag{9}$$

It should be noted that $S^{(1)} = \mathbf{0}$. Let $e(k) = [e^{(1)}(k)^\top, \ldots, e^{(m)}(k)^\top]^\top$. By (9), we write the compact form of the vehicle platoon's driving state estimation error dynamics as follows:

$$e(k+1) = \underbrace{\begin{bmatrix} R^{(1)} & & & \\ S^{(2)} & R^{(2)} & & \\ & \ddots & \ddots & \\ & & S^{(m)} & R^{(m)} \end{bmatrix}}_{U} e(k). \tag{10}$$

Since the system matrix U in (10) is lower triangular, it suffices to prove that all $R^{(j)}$ are Schur stable. The observability of $(A, C_{j,j})$ for each $j \in \mathrm{V}$ ensures that an appropriate gain F_j can be chosen to make the matrix $A - F_j C_{j,j}$ Schur stable, i.e., $\rho(A - F_j C_{j,j}) < 1$. In addition, when the matrix $Q^{(j)}$ is Schur stable, indicated by a strongly connected intervehicle network according to Lemma 2, combining the fact that $\mathrm{sp}(A) = \{1, 1, 1 - \frac{\tau_s}{\tau}\}$ for each $j \in \mathrm{V}$, $Q^{(j)} \otimes A$ is Schur stable. Therefore, we obtain that $\lim_{k\to\infty} e(k) = 0$, i.e., $\lim_{k\to\infty} e^{(j)}(k) = 0$, which completes the proof. ■

So far, question *Q1* has been answered. According to the conditions in Theorem 1, we can find the appropriate gain parameters to make the output of the proposed observer (5) asymptotically converge to the real driving state of the platoon. It should be noted that the results of Theorem 1 are based on the condition that the input $u_i(k)$ is known to all vehicles for all $i \in \mathrm{V}$. In general, control inputs of other vehicles $u_j(k), j \neq i$, are not available for vehicle i. In this case, we can regard u_j as a bounded disturbance, and the bound is large enough to incorporate all possible values of the input of vehicle j, i.e., $\|u_j\| < \beta$, for $\forall j \in \mathrm{V}$. Then, by a similar analysis to Theorem 1, it can be proved that, under the proposed observer (5), where $u_j(k) = \mathbf{0}$ in (5b), the entire estimation error will be bounded by a quantity that scales with the bound β. Theorem 1 is given for the routine operation of a platoon, i.e., no merging or splitting.

For the case of merging or splitting, the main results are provided in Section IV.

Remark 1: From Theorem 1, each vehicle can independently determine its local observer's parameters, including Luenberger observer gain and consensus weights, implying that observer (5) is constructed in a fully distributed manner. Additionally, the convergence time of the observer (5) depends on the Luenberger observer gain and the communication network topology. Furthermore, the proposed observer can be applied to heterogeneous platoons easily, i.e., each vehicle only needs to utilize an adaptive observer developed in [29] to learn the system matrix of other vehicles.

IV. Plug-and-Play Operations in Open Platoons

In this section, we demonstrate the effectiveness of our observer when handling vehicle arrivals and departures. We introduce two algorithms that enable plug-and-play operations of the observer, and we also perform a stability analysis of the estimation error dynamics in both scenarios. Additionally, we make an assumption about the communication network, which we describe as follows.

Assumption 2: The communication network of the open platoon can always maintain strongly connected whenever new vehicles join or existing ones leave the platoon.

Assumption 2 can be fulfilled using various widely employed communication topologies in vehicle platoon scenarios, such as the undirected leader-to-all topology, k-nearest neighbor topology, and k-nearest neighbor ring topology [30]. These communication topologies possess the capability to sustain a strongly connected network when a small number of vehicles join or depart from the platoon.

A. Distributed State Estimation for Platoon Merging

We consider the case that a new vehicle with a model satisfying (2) wishes to join a platoon comprising of m vehicles. For ease of understanding, we refer to this new vehicle as vehicle $m+1$. Before joining the platoon, vehicle $m+1$ analyzes real-time traffic conditions and selects a suitable position to join the platoon. At time step t_I, it sends a request to the vehicles ahead and behind near the entry point. The merging process takes place between time t_I and t_I+T, during which all vehicles prepare for the platoon merging. During this period, all vehicles in set V are informed about the joining of vehicle $m+1$ via the communication network G and allocate additional computation and storage resources for the estimation of the new vehicle. The vehicles adjacent to the vehicle $m+1$ coordinate with it and prepare to renew the communication network to include the new member. Once the technical preparation is completed, the entire system switches to a new vehicle platoon with the new communication network $\tilde{\mathrm{G}} = (\tilde{\mathrm{V}}, \tilde{\mathrm{E}})$, where $\tilde{\mathrm{V}} = \{1, \ldots, m+1\}$ and $\tilde{\mathrm{E}} \subseteq \tilde{\mathrm{V}} \times \tilde{\mathrm{V}}$. We denote the in-neighbors and out-neighbors of node i in graph $\tilde{\mathrm{G}}$ as $\tilde{\mathrm{N}}_i$ and $\tilde{\mathrm{S}}_i$, respectively. In this scenario of platoon merging, the proposed distributed driving state observer is described in detail in Algorithm 1.

In the following discussion, we examine the stability of the estimation error dynamics of the proposed observer in the context of platoon merging. We use the symbols $\tilde{w}_{i0}^{(j)}$ and $\tilde{w}_{il}^{(j)}$ to

Algorithm 1: Distributed Observer for Platoon Merging.

Input: $\mathrm{G} = (\mathrm{V}, \mathrm{E})$, $(A, C_{i,i})$, $\mathbf{x}_i(0) \in \mathbb{R}^{3m}$ and $\hat{\mathbf{x}}_i(0) \in \mathbb{R}^{3m}$ set to arbitrary value, $\{F_i, w_{i0}^{(j)}, w_{il}^{(j)}\}_{i,j\in \mathrm{V}, l\in \mathrm{N}_i}$ chosen by Theorem 1, t_I, T
for $d = t_I, \ldots, k+1$ **do**
  for $i \in \tilde{\mathrm{V}}$ **do**
    if $d < t_I + T$ and $i \in \mathrm{V}$ **then**
      Receive $\hat{\mathbf{x}}_l(k) \in \mathbb{R}^{3m}$ from vehicle l in set N_i
      Calculate $\hat{\mathbf{x}}_i(k) \in \mathbb{R}^{3m}$ based on (5)
      Receive the entry request and make preparation
    if $d < t_I + T$ and $i = m+1$ **then**
      Send the entry request and make preparation
    if $d \geq t_I + T$ and $i = m+1$ **then**
      Receive $\hat{\mathbf{x}}_l(k) \in \mathbb{R}^{3(m+1)}$ from vehicle l in set $\tilde{\mathrm{N}}_{m+1}$
      Calculate $\hat{\mathbf{x}}_{m+1}(k) \in \mathbb{R}^{3(m+1)}$ based on (5)
    if $d \geq t_I + T$ and $i \in \tilde{\mathrm{S}}_{m+1}$ **then**
      Receive $\hat{\mathbf{x}}_l(k) \in \mathbb{R}^{3(m+1)}$ from l in set $\tilde{\mathrm{N}}_i$
      Renew the consensus weights to $\{\tilde{w}_{i0}^{(j)}, \tilde{w}_{il}^{(j)}\}_{j\in\tilde{\mathrm{V}}, l\in\tilde{\mathrm{N}}_i}$
      Calculate $\hat{\mathbf{x}}_i(k) \in \mathbb{R}^{3(m+1)}$ based on (5)
    else
      Keep the original update unchanged
      Add the estimate $\hat{x}_i^{(m+1)}(k)$ calculated based on (5b)
    end if
  end for
end for
Output: $\hat{\mathbf{x}}_i(k+1) \in \mathbb{R}^{3(m+1)}$, $\tilde{\mathrm{G}} = (\tilde{\mathrm{V}}, \tilde{\mathrm{E}})$

represent the redesigned consensus weights in the new vehicle platoon. To facilitate our analysis, we introduce the matrix $\tilde{Q}^{(j)} \in \mathbb{R}^{(m+1)\times(m+1)}$, which corresponds to $Q^{(j)}$ in (6). As we assume that the graph $\tilde{\mathrm{G}}$ is strongly connected, we can apply Lemma 2 to show that $\tilde{Q}^{(j)}$ is Schur stable. Importantly, we find that the joining of vehicle $m+1$ does not impact the lower triangular structure of the matrices $\tilde{U}$ and $\tilde{R}^{(j)}$. These matrices correspond to U and $R^{(j)}$ in (10) and (9), respectively. Therefore, based on the proof of Theorem 1, we obtain the main results as follows.

Proposition 1: Consider a new vehicle modeled as (2) joins the platoon and a new platoon forms with a communication network $\tilde{\mathrm{G}} = (\tilde{\mathrm{V}}, \tilde{\mathrm{E}})$ satisfying Assumption 2. Then for each vehicle $i \in \tilde{\mathrm{V}}$, there exists a choice of F_i, $\{\tilde{w}_{i0}^{(j)}, \tilde{w}_{il}^{(j)}\}_{j\in\tilde{\mathrm{V}}, l\in\tilde{\mathrm{N}}_i}$, such that the estimate $\hat{\mathbf{x}}_i(k) \in \mathbb{R}^{3(m+1)}$ updated by (5) asymptotically converges to the new platoon's real driving state $\tilde{\mathbf{x}}(k) = [x_1(k)^\top, \ldots, x_{m+1}(k)^\top]^\top \in \mathbb{R}^{3(m+1)}$, i.e., $\lim_{k\to\infty} \|\hat{\mathbf{x}}_i(k) - \tilde{\mathbf{x}}(k)\| = 0$.

B. Distributed State Estimation for Platoon Splitting

We consider the case that vehicle $r \in \mathrm{V}$ intends to leave a platoon of m vehicles that includes itself and that the platoon is connected by a communication network G. Prior to departure, vehicle r sends a departure request to its out-neighbors. Through

Algorithm 2: Distributed Observer for Platoon Splitting.

Input: $\mathrm{G} = (\mathrm{V}, \mathrm{E})$, $(A, C_{i,i})$, $\mathbf{x}_i(0) \in \mathbb{R}^{3m}$ and $\hat{\mathbf{x}}_i(0) \in \mathbb{R}^{3m}$ set to arbitrary value, $\{F_i, w_{i0}^{(j)}, w_{il}^{(j)}\}_{i,j\in\mathrm{V},l\in\mathrm{N}_i}$ chosen by Theorem 1, t_O
for $d = t_O, \ldots, k+1$ **do**
  Form a new platoon with the network $\breve{\mathrm{G}} = (\breve{\mathrm{V}}, \breve{\mathrm{E}})$
  for $i \in \breve{\mathrm{V}}$ **do**
    if $i \in \mathrm{S}_r$ **then**
      Receive $\hat{\mathbf{x}}_l(k) \in \mathbb{R}^{3(m-1)}$ from l in set $\breve{\mathrm{N}}_i$
      Renew the consensus weights to $\{\breve{w}_{i0}^{(j)}, \breve{w}_{il}^{(j)}\}_{j\in\breve{\mathrm{V}},l\in\breve{\mathrm{N}}_i}$
      Calculate $\hat{\mathbf{x}}_i(k) \in \mathbb{R}^{3(m-1)}$ based on (5)
    else
      Keep the original update unchanged
      Delete all information related to vehicle r
    end if
  end for
end for
Output: $\hat{\mathbf{x}}_i(k+1) \in \mathbb{R}^{3(m-1)}$, $\breve{\mathrm{G}} = (\breve{\mathrm{V}}, \breve{\mathrm{E}})$

information exchange among the remaining vehicles in set V/r, all vehicles are informed about the departure directly or indirectly. Then, the state estimation and information transmission of vehicle r are stopped, and all data associated with vehicle r is deleted. Besides, vehicles adjacent to vehicle r need to cooperate with each other to ensure that a new vehicle platoon is formed when vehicle r leaves. Suppose vehicle r departs at time step t_O. At the same time, a new platoon is formed with a new communication network $\breve{\mathrm{G}} = (\breve{\mathrm{V}}, \breve{\mathrm{E}})$, where $\breve{\mathrm{V}} = \mathrm{V}/r$. For node i in graph $\breve{\mathrm{G}}$, let $\breve{\mathrm{N}}_i$ and $\breve{\mathrm{S}}_i$ denote its in-neighbors and out-neighbors, respectively. The proposed distributed driving state observer for this scenario is described in detail in Algorithm 2, where $\breve{w}_{i0}^{(j)}$ and $\breve{w}_{il}^{(j)}$ are the redesigned consensus weights. Similar results to Proposition 1 can be obtained through the same analysis and are omitted for brevity.

It is possible for more than one vehicle to join a platoon, either one by one or at the same time. If vehicles join one by one, the platoon merging problem can be solved by repeating the procedure outlined for platoon merging. If vehicles join at the same time, the proposed distributed observer must adapt to the number of joining vehicles. It is important to note that the local observer redesign does not propagate further in the communication network. Once the platoon composition stops evolving, the stability analysis of the resulting observer is similar to that for platoon merging. Therefore, this analysis is omitted for brevity. Similar results can be seen in the case of the observer against platoon splitting. In summary, the proposed distributed observer (5) scales well in open platoons and can be easily operated with plug-and-play functionality, as shown in Algorithms 1 and 2. This answers question *Q2*. We emphasize the plug-and-play operation of the proposed observer (5), particularly focusing on its performance before and after the merging (or splitting) event. Other cooperative maneuvers, including gap-creating, lane-changing, or overtaking strategies involved in the merging and splitting scenarios, can be found in [7], [8], [9], [10]. We plan to investigate the impact of the proposed observer (5) on these cooperative maneuvers in future research. Besides, in the aforementioned papers, the merging and splitting process of the platoon is modeled as a switched system for analysis. How to design a plug-and-play distributed driving state observer in this modeling method is a promising problem, which is left as our future work.

Remark 2: The secure distributed driving state estimator in [12] ensures that each vehicle can estimate the global driving state in the presence of attacks. It assumes that the vehicle platooning system is observable for each vehicle and thus it inherently functions as a centralized observer, which prevents its application in open vehicle platoons. Differently, our observer (5) is proposed for each vehicle to estimate the global driving state of the platoon with joint observability, which is more relaxed than the observability assumption used in [12]. Additionally, the distributed observer for continuous-time LTI systems in [31] used an adaptive consensus scheme to determine the parameters of the proposed observer locally, thereby enabling the plug-and-play operation. For the proposed observer (5), its parameters including the Luenberger observer gain and consensus gains, can be locally and directly determined by each vehicle. This benefits from both the extended-state framework and the nature of the platooning system model.

V. Simulation Results

In this section, we validate the theoretical results and investigate the driving state estimation performance through extensive simulations. Specifically, we apply our distributed driving state observer (5) to an open 2-nearest neighbor platoon. Moreover, combining the proposed observer (5) with the results of [15], we propose an distributed observer-based controller and provide a numerical simulation to illustrate its performance.

A. Distributed Driving State Estimation for an Open 2-Nearest Neighbor Platoon

We apply the proposed observer (5) to an open 2-nearest neighbor platoon consisting of four homogeneous vehicles. Each vehicle can transmit its estimate of the platoon driving state to its two preceding vehicles and two following vehicles. We set the engine time constant $\tau = 1$ and the sampling period $\tau_s = 0.02$. The initial driving state of each vehicle is set to $x_1(0) = [150, 30, 0]^\top$, $x_2(0) = [123, 25, 2.1]^\top$,$x_3(0) = [92, 27, 2.9]^\top$, $x_4(0) = [60, 29, 2.4]^\top$. The initial estimates of all vehicles are set to $\mathbf{0}$. By Theorem 1, the local Luenberger observer gains are chosen as

$$F_1 = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 1 \end{bmatrix}, F_2 = F_3 = F_4 = \begin{bmatrix} 0.2 & 1 & 0 \\ 0 & 0 & 0.9 \\ 0.5 & 0.5 & 0 \end{bmatrix}$$

all together satisfying $A - F_i C_{i,i}$ being Schur stable. Considering the constraint of consensus weights in Theorem 1, we use the Metropolis weights design scheme [32] to assign the weights of data fusion in (5b) for all the following simulations, where for

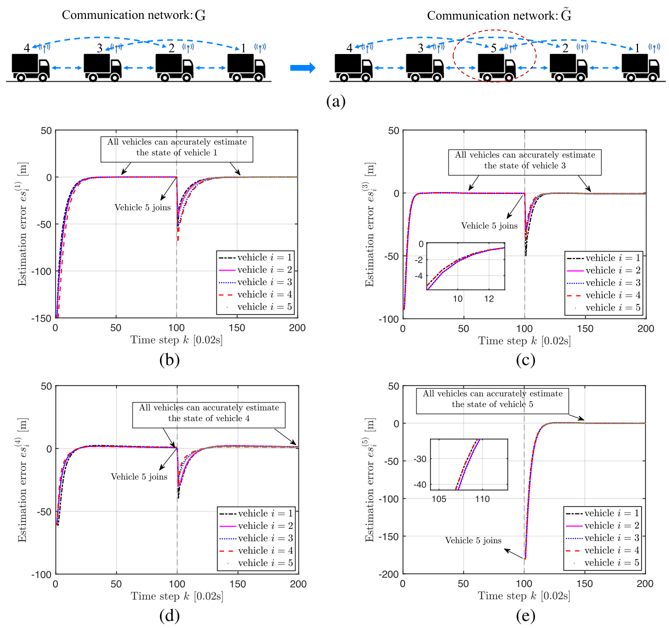


Fig. 3. Estimation error for the positions of vehicles 1, 3, 4, and 5 by all vehicles in platoon merging scenario. (a) Schematic diagram of platoon merging case. (b) Estimation for vehicle 1. (c) Estimation for vehicle 3. (d) Estimation for vehicle 4. (e) Estimation for vehicle 5.

$i \in \mathrm{V}$, the consensus weight $w_{il}^{(j)} = \frac{1}{d_i^{\mathrm{G}(j)}+1}$ if $l \in \mathrm{N}_i^{\mathrm{G}(j)} \cup \{i\}$, and $w_{il}^{(j)} = 0$ otherwise. Based on the above setup, we introduce examples of the proposed distributed driving state observer in the platoon merging and splitting scenario.

Merging scenario: We begin by validating the case of platoon merging, where vehicle 5 joins the 2-nearest neighbor platoon mentioned earlier. At time step t_I, vehicle 5 sends an entry request to vehicles 2 and 3. In the next T time period, all vehicles need to complete tasks including control strategies adjusting and communication links renewing to form a new platoon. Then, at time step $t_I + T = 100$, the entire system switches to a new vehicle platoon consisting of five vehicles, where vehicle 5 becomes a member and the schematic diagram is shown in Fig. 3. The initial driving state of vehicle 5 is set as $x_5(100) = [180, 28, 2.3]^\top$. In the new platoon, with the communication network $\tilde{\mathrm{G}}$ shown in Fig. 3(a), vehicle 5 uses the proposed observer (5) to obtain the driving state estimates of the whole platoon. The local Luenberger gain F_5 is chosen to be the same as F_2, and the consensus weights are determined by using the Metropolis weights. To ensure that the estimate converges to the actual driving state, the vehicles in set $\tilde{\mathrm{S}}_5 = \{1, 2, 3, 4\}$ need to redesign their consensus weights according to the new neighborship in $\tilde{\mathrm{G}}$ by the Metropolis weights design scheme. Other parameters remain unchanged. Note that the redesign of these parameters is completed by each vehicle independently assigning an appropriate weight to the information on each communication link with its in-neighbors. Fig. 3(b)–(e) shows the estimation error dynamics for the positions of vehicles 1, 3, 4, and 5 during the platoon merging scenario, as observed by all vehicles. It can be seen that vehicles 1–5 begin estimating the driving state of the new platoon at time step 100. We observe that all estimation error dynamics converge quickly to zero after vehicle 5 joins the vehicle platoon, which validates the effectiveness of our approach against platoon merging.

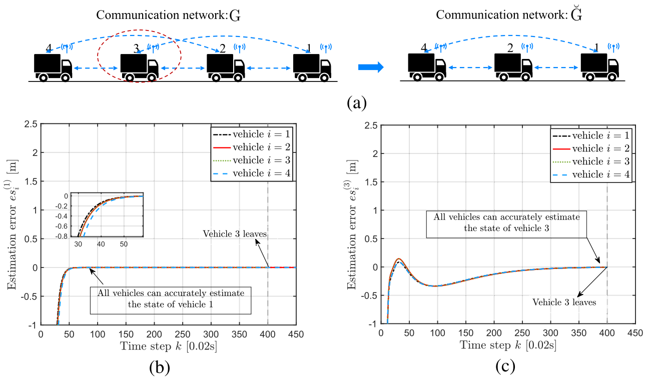


Fig. 4. Estimation error for the positions of vehicles 1 and 3 by all vehicles in the platoon splitting scenario. (a) Schematic diagram of platoon splitting case. (b) Estimation for vehicle 1. (c) Estimation for vehicle 3.

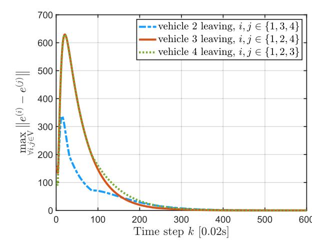


Fig. 5. Evolution of the deviation $\max_{\forall i,j \in \mathrm{V}} \|e^{(i)}(k) - e^{(j)}(k)\|$.

Splitting scenario: In the platoon splitting scenario, we consider that vehicle 3 leaves the original 2-nearest neighbor platoon. First, vehicle 3 sends a departure request to vehicles 1, 2, and 4. When the remaining vehicles are informed via multihop information, they stop the state estimation and remove the information of vehicle 3. Moreover, they need to cooperate with each other to form a new platoon. Assuming that vehicle 3 leaves the platoon at time step $t_O = 400$, a new platoon with three vehicles is established. The communication network of this new platoon denoted as $\breve{\mathrm{G}}$ is illustrated in Fig. 4(a). To ensure the stable operation of the proposed distributed observer, the vehicles in set $\mathrm{S}_3 = \{1, 2, 4\}$ need to redesign their consensus weights based on the new communication network $\breve{\mathrm{G}}$ using Metropolis weights. All other parameters remain unchanged. Note that vehicles 1, 2, 4 can complete this renewal of consensus weights independently. Fig. 4(b)–(c) depicts the estimation error dynamics for the positions of vehicles 1 and 3. It is evident that vehicles 1, 2, 4 do not estimate the driving state of vehicle 3 from time step 400 onward (i.e., the green dotted line disappears from time step 400). Moreover, the estimation error dynamics are consistently stable throughout the process. Meanwhile, we investigate the evolution of the deviation $\max_{\forall i,j \in \mathrm{V}} \|e^{(i)}(k) - e^{(j)}(k)\|$ for the departure of all vehicles except the leading vehicle, as shown in Fig. 5. It can be observed that all the estimation error dynamics

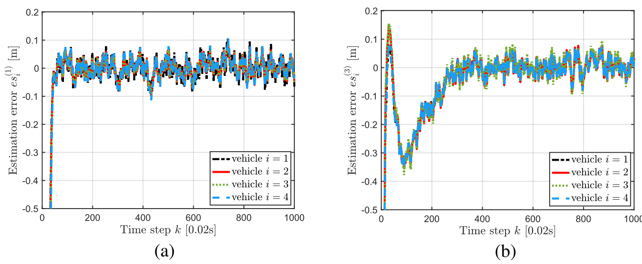


Fig. 6. Performance in the presence of noise. (a) Estimation for vehicle 1. (b) Estimation for vehicle 3.

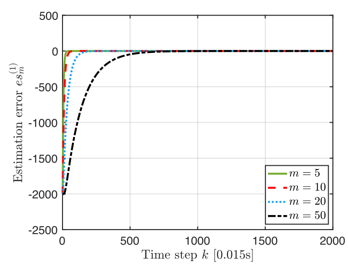


Fig. 7. Performance in different sizes of platoons.

always converge to zero no matter which vehicle other than the leading vehicle leaves the platoon.

Additionally, we conduct a simulation to demonstrate the performance of the proposed observer (5) in the presence of noise. In this simulation, we still consider the abovementioned 2-nearest neighbor platoon consisting of four vehicles, where each vehicle $i \in \{1, 2, 3, 4\}$ is modeled as a discrete linear state-space model with process noise $\omega_i(k)$ and measurement noise $\nu_i(k)$. Both types of noise are bounded, with a bound of 0.05. Note that the only difference between this model and the original vehicle model (2)–(3) lies in the addition of process noise $\omega_i(k)$ to (2) and measurement noise $\nu_i(k)$ to (3). All other simulation parameters remain consistent with those specified previously. The simulation results are shown in Fig. 6. It can be observed that the estimation error dynamics are bounded, which verifies the effectiveness of our observer in the presence of noise.

Furthermore, we conduct simulations to demonstrate the performance of the proposed observer (5) for different sizes of platoons. We consider four platoons with different sizes, i.e., $m = 5, 10, 20, 50$, where the platoon with five vehicles is a 5-nearest neighbor platoon and the others are 9-nearest neighbor platoons. In Fig. 7, we depict the position estimation error dynamics of the leading vehicle by the last one in each platoon. It can be seen that the estimation error dynamics converge quickly to zero for all platoons.

B. Distributed State Estimation for Platoon Control

Here, to answer question *Q3*, we focus on using the results of the proposed observer to improve longitudinal control performance for the line graph platoon. The constant time headway spacing (CTHS) policy is adopted to maintain a small relative distance between neighboring vehicles. The desirable distance

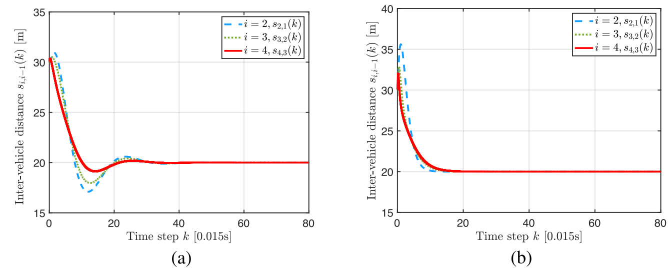


Fig. 8. Performance comparison between the controller proposed in [15] and our observer-based controller (12). (a) Inter-vehicle distances in [15]. (b) Inter-vehicle distances in this paper.

$d_{i,i-1}$ between vehicle i and its preceding vehicle $i-1$ is

$$d_{i,i-1}(k) = d + hv_i(k),\ \forall i \in \{2, 3, 4\} \tag{11}$$

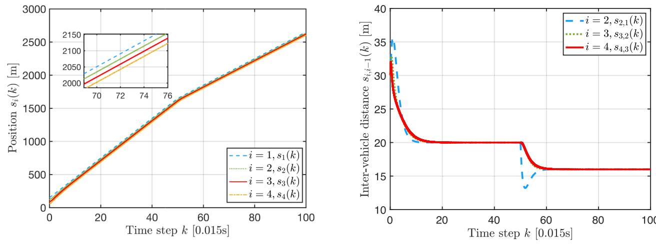
where d denotes the constant distance and $h > 0$ denotes the constant headway time. In this simulation, we focus on the internal stability to evaluate the controller's performance, where the internal stability describes the stability of platoons without disturbances. Combining our observer and the controller of [14], [15], we propose the following distributed observer-based controller to control the line graph platoon. For each vehicle $i \in \{2, 3, 4\}$, the control input $u_i(k)$ is

$$\begin{aligned} u_i(k) = \sum_{j=1}^{i-1} &(\kappa_s(\hat{s}_i^{(j)}(k) - s_i(k) - d_{i,j}(k)) \\ &+ \kappa_v(\hat{v}_i^{(j)}(k) - v_i(k)) + \kappa_a(\hat{a}_i^{(j)}(k) - a_i(k))) \end{aligned} \tag{12}$$

where $\hat{s}_i^{(j)}(k)$, $\hat{v}_i^{(j)}(k)$, and $\hat{a}_i^{(j)}(k)$ are elements of the estimate $\hat{x}_i^{(j)}(k)$. The proposed controller (12) enables each vehicle to control itself utilizing estimates of the driving states of all preceding vehicles acquired locally by itself using the proposed observer (5).

We conduct a group of comparative experiments on the performance of our distributed observer-based controller (12) and the controller proposed in [15], where each vehicle uses the driving state transmitted by its neighbor vehicles, i.e., the one in front and the one behind. We use the same parameters as those in [15], i.e., $\tau_s = 0.015, h = 0.4, d = 8, \tau = 0.01, \kappa_s = 0.45, \kappa_v = 1, \kappa_a = -0.2$. The observer parameters are the same as in Section V-A. Besides, the initial positions, velocities, and accelerations of four vehicles are set as $x_1(0) = [150, 30, 0]^\top, x_2(0) = [120, 29, 2.1]^\top, x_3(0) = [90, 29.5, 2.6]^\top, x_4(0) = [60, 26, 2.3]^\top$.

The simulation results depicted in Fig. 8 illustrate the intervehicle distance $s_{i,i-1}(k)$ between adjacent vehicles $i-1$ and i, where $i \in \{2, 3, 4\}$. One can observe that our proposed distributed observer-based controller (12) makes the intervehicle distances converge to a constant value $hv_1(0) + d$ (i.e., 20 m), which satisfies the CTHS policy. Meanwhile, our observer-based controller (12) results in faster consensus on velocity and acceleration than the controller proposed in [15].



(a) (b)

Fig. 9. Performance of controller (12) under the leading vehicle braking. (a) Positions of all vehicles. (b) Inter-vehicle distances.

We further verify the effectiveness of controller (12) in an emergency situation, i.e., sudden braking of the leading vehicle. Specifically, the leading vehicle brakes abruptly at the 50th second, reducing its speed to $20\,\mathrm{m/s}$. As shown in Fig. 9, the platoon quickly regained its internal stability, with all spacing errors converging to the new desired value of 16. These results indicate that our observer-based controller is capable of effectively handling unexpected events like emergency braking.

The computational burden of our distributed observer-based controller (12) arises from the calculation of (5) and (12). Note that the formulas of the proposed observer (5) and controller (12) only involve low-dimensional (3×3) matrices, low-dimensional (3×1) vectors, and scalars. As a result, the proposed controller (12) has a low computational burden. Furthermore, the utilization of efficient data structures and optimization techniques enables real-time implementation of our controller even in scenarios with a large number of vehicles. On the other hand, in our design, the only information that needs to be transmitted over the communication network for each vehicle is its estimate of the global driving state of the platoon, represented as a vector with a dimension of $3m$. Practically, this vector corresponds to the maximum amount of transmitted data of $24m$ Bytes, which is significantly lower than the bandwidth provided by the current C-V2X communication technology used in vehicle platoons. This holds true even for a vehicle platoon with a large number of vehicles. As a result, the proposed controller (12) enables real-time information exchange without imposing a heavy communication burden on the platoon.

VI. Conclusion

This article developed a novel approach to distributed driving state estimation for open vehicle platoons. By leveraging the decoupled and locally observable nature of driving states in platoons, we proposed a distributed driving state observer that follows the leader–follower consensus method. To ensure the stability of our proposed observer, we derived the necessary and sufficient conditions, and analyzed its flexibility. We showcased the observer's plug-and-play operations in an open platoon, which benefits from both the platooning system model and the extended-state framework of the observer. Our work highlights that exploiting the output of the proposed observer can significantly improve platoon control performance. Furthermore, it is important to investigate the potential of this approach in solving critical problems, such as safety, fuel optimization, attack detection, and secure control for vehicle platooning. Extending the plug-and-play design for the distributed observer to general networked linear systems is also an important research issue that beckons further investigation.

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