USEFUL FORMULAE AND OTHER INFORMATION

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$$

x	$\cos x$	$\sin x$
0	1	0
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	0	1
$\frac{3\pi}{4}$	$-\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
π	-1	0

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j(2\pi/T)kt}$$

$$c_k = \frac{1}{T}\int_T x(t)e^{-j(2\pi/T)kt}dt$$

$$S(\omega) = \frac{\omega_s}{2\pi}\sum_{k=-\infty}^{\infty} X(\omega - k\omega_s)$$

$$\mathscr{F}x(\omega) = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$\mathscr{F}^{-1}X(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega$$

$$H(\omega) = \frac{2\pi}{\omega_s}\,\mathrm{rect}\left(\frac{\omega}{\omega_s}\right)$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k \delta(\omega - k\omega_0)$$

$$X(\omega) = \sum_{k=-\infty}^{\infty} \omega_0 X_T(k\omega_0)\delta(\omega - k\omega_0)$$

$$a_k = \frac{1}{T}X_T(k\omega_0)$$

Fourier Transform Properties

Property	Time Domain	Frequency Domain
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(\omega) + a_2X_2(\omega)$
Time-Domain Shifting	$x(t-t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency-Domain Shifting	$e^{j\omega_0 t}x(t)$	$X(\omega - \omega_0)$
Time/Frequency-Domain Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time-Domain Convolution	$x_1 * x_2(t)$	$X_1(\omega)X_2(\omega)$
Frequency-Domain Convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1 * X_2(\omega)$
Time-Domain Differentiation	$\frac{d}{dt}x(t)$	$j\omega X(\omega)$
Frequency-Domain Differentiation	$tx(t)$	$j\frac{d}{d\omega}X(\omega)$
Time-Domain Integration	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$
Parseval's Relation	$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2 d\omega$	

Fourier Transform Pairs

Pair	$x(t)$	$X(\omega)$
1	$\delta(t)$	1
2	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$
3	1	$2\pi\delta(\omega)$
4	$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$
5	$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$
6	$\cos(\omega_0 t)$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
7	$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
8	$\mathrm{rect}\left(\frac{t}{T}\right)$	$\lvert T\rvert\,\mathrm{sinc}\left(\frac{T\omega}{2}\right)$
9	$\mathrm{sinc}(Bt)$	$\frac{\pi}{\lvert B\rvert}\,\mathrm{rect}\left(\frac{\omega}{2B}\right)$
10	$e^{-at}u(t)$, $\mathrm{Re}\{a\} > 0$	$\frac{1}{a+j\omega}$
11	$t^{n-1}e^{-at}u(t)$, $\mathrm{Re}\{a\} > 0$	$\frac{(n-1)!}{(a+j\omega)^n}$
12	$e^{-at}\cos(\omega_0 t)u(t)$, $\mathrm{Re}\{a\} > 0$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$
13	$e^{-at}\sin(\omega_0 t)u(t)$, $\mathrm{Re}\{a\} > 0$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$
14	$e^{at}u(-t)$, $\mathrm{Re}\{a\} > 0$	$\frac{1}{a-j\omega}$